Physics 209: Assignment #6

The questions that follow are intended to give you some practice in reading and interpreting the space-time diagrams developed in Part 10 of the lecture notes. All questions refer to the accompanying Figures.

Figures 1-4, 6, and 7 show lines of constant time and position used by Alice and Bob to interpret the events in a space-time diagram. The number of feet between two events on different lines of constant position (in the frame of the person using those lines) is the difference between the numbers that label the lines (which it is convenient to refer to as "the 1-foot line", "the 2-foot line", etc., according to their label.) Similarly the number of nanoseconds between two events on different lines of constant time is the difference between the numbers that label the lines (which we can call the "1-nanosecond line", the "2-nanosecond line", etc.)

Alice uses the vertical lines of constant position (labeled 1-6 in vertical (roman) type) and the horizontal lines of constant time (labeled 1-6 in vertical type). The figure also shows some of Alice's intermediate lines (that would be labeled 1.25, 1.5, 1.75, etc., but have been left unlabeled so as not to clutter up the figure.)

Bob uses the slanting lines of constant position (labeled 1-8 in heavy slanting (italic) numerals) and the slanting lines of constant time (labeled 1-9 in light slanting numerals). (I have not complicated the figure by adding any additional lines for Bob.)

I.

Here are some short questions about Figures 1-3. Your answers should use feet (f) for the unit of distance and nanoseconds (ns) for the unit of time, with speeds in units of feet per nanosecond. (The speed of light is $c = 1$ f/ns.) Your answers can be brief, but they should make it clear to one who was perplexed by the question, how you extracted your answer from the figure.

1. What is the velocity of the Bob's frame of reference with respect to Alice's?

2. Figure 2 shows two events A and B (the large black dots). What are the distance D_A and and time T_A between the events according to Alice? What are the distance D_B and and time T_B between the events according to Bob? Your answer should make it clear how you acquired this information from the diagram.

3. Confirm from the numbers you gave in answer to question 2 that the *interval* between the events, $|c^2T^2 - D^2|$ is the same whether it is computed from Alice's or Bob's time and distance.

4. The heavy line in Figure 3 is the space-time trajectory of a moving particle. What is the velocity of the particle in Alice's frame? What is its velocity in Bob's frame? (For your convenience, I have indicated with big black dots two events on the trajectory of

the particle whose positions and times are particularly clearly related to both Alice's and Bob's lines of constant time and position.) Your answer should make it clear how you acquired this information from the diagram.

5. Confirm (arithmetically) that your answers to questions 4 and 1 are consistent with the relativistic velocity addition law

$$w = \frac{u+v}{1+uv/c^2}$$

stating explicitly what it is that is moving with respect to what at each of the velocities w, u, and v.

II.

The heavy line in Figure 4 is the space-time trajectory of a wonderful faster-than-light rocket that Alice has invented. Prior to the event A the rocket is stationary on its launching pad in Alice's frame. At event A the rocket is fired to the right at a speed faster than light. (How fast is it going in Alice's frame?) At event B Alice's rocket crashes into a cliff. After event B it is a stationary (in Alice's frame) wreck.

Write a brief description of the entire history of this rocket according to Bob. The history should start at a time long before the launching and continue on to a time long after the crash, and it should include everything that happens between the crash and the launching. Expect some parts of Bob's history to be quite strange.

In the history you write for Bob feel free to refer to an event that lies on Bob's 5-nanosecond line (for example) as an event that takes place at the time 5 nanoseconds. (And an event that takes place on Bob's 3-foot line can be refered to as taking place at the 3-foot position.)

III.

Write a two-part caption for Figure 5 that enables somebody familiar with the general principles behind Minkowski's space-time diagrams to see that the figures can be interpreted either as (a) an illustration of the $T = Dv/c^2$ rule for simultaneous events or (b) the $T = Dv/c^2$ rule for synchronized clocks. You can call the frame using horizontal and vertical lines of constant time and position Alice's, and the frame moving to the right in Alice's frame with speed v, Bob's. Be sure in part (a) to state what the events are and in whose frame they are simultaneous. Be sure in part (b) to state what the clocks are and in whose frame they are synchronized.

IV. *OPTIONAL*

Figures 6 and 7 contain two simple, alternative, proofs that the area of Bob's unit rhombi are the same as the area of Alice's.[1] All you need to know is that a "unit rhombus"

[1] This point is demonstrated in the Appendix to Part 10 of the lecture notes, in a less simple, but completely general way . The arguments implicit in Figures 6 and 7 should make sense even if you had difficulty with or chose not to read the argument in the lecture notes.

(plural, "rhombi") in either frame is a region bounded by lines of constant time and position whose labels differ by one nanosecond and one foot. Thus Alice's unit rhombi are simply squares (made up of 16 of the little squares. Bob's are parallelograms, with four equal sides.)

(a) Write a caption for Figure 6 that explains how you can see from the subfigure with heavy lines at the top that Alice and Bob's unit rhombi (the two subfigures with heavy lines at the bottom) do indeed have the same area.[2] (Once you understand what I'm asking you to show, nothing more is involved than figuring out the areas of the two heavy squares, and the three right triangles contained in the bigger square.)

(b) Write a caption for Figure 7 that makes it clear why the figure with the heavy solid and dashed lines also establishes that Alice's and Bob's unit rhombi have the same area. (This time nothing more is involved than examining the comparative lengths of the dashed diagonals of the square and the rhombus, and thinking a bit about how those lengths are related to the relevant areas.)

[2] As usual, the test for a satisfactory caption is that it ought to make it clear for somebody who is baffled by the figure (as you were initially) why the figure performs as advertised.

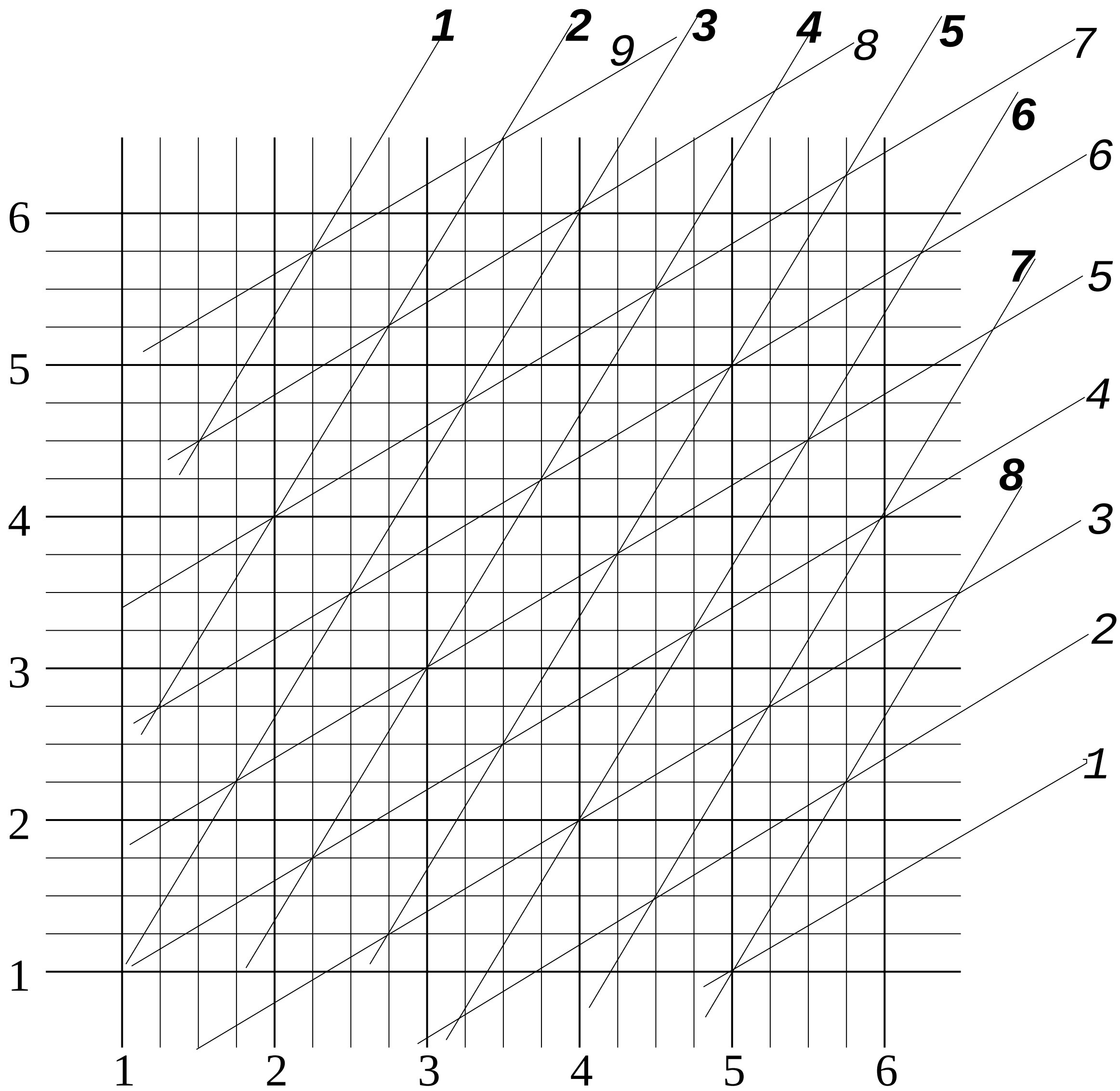

Figure 1

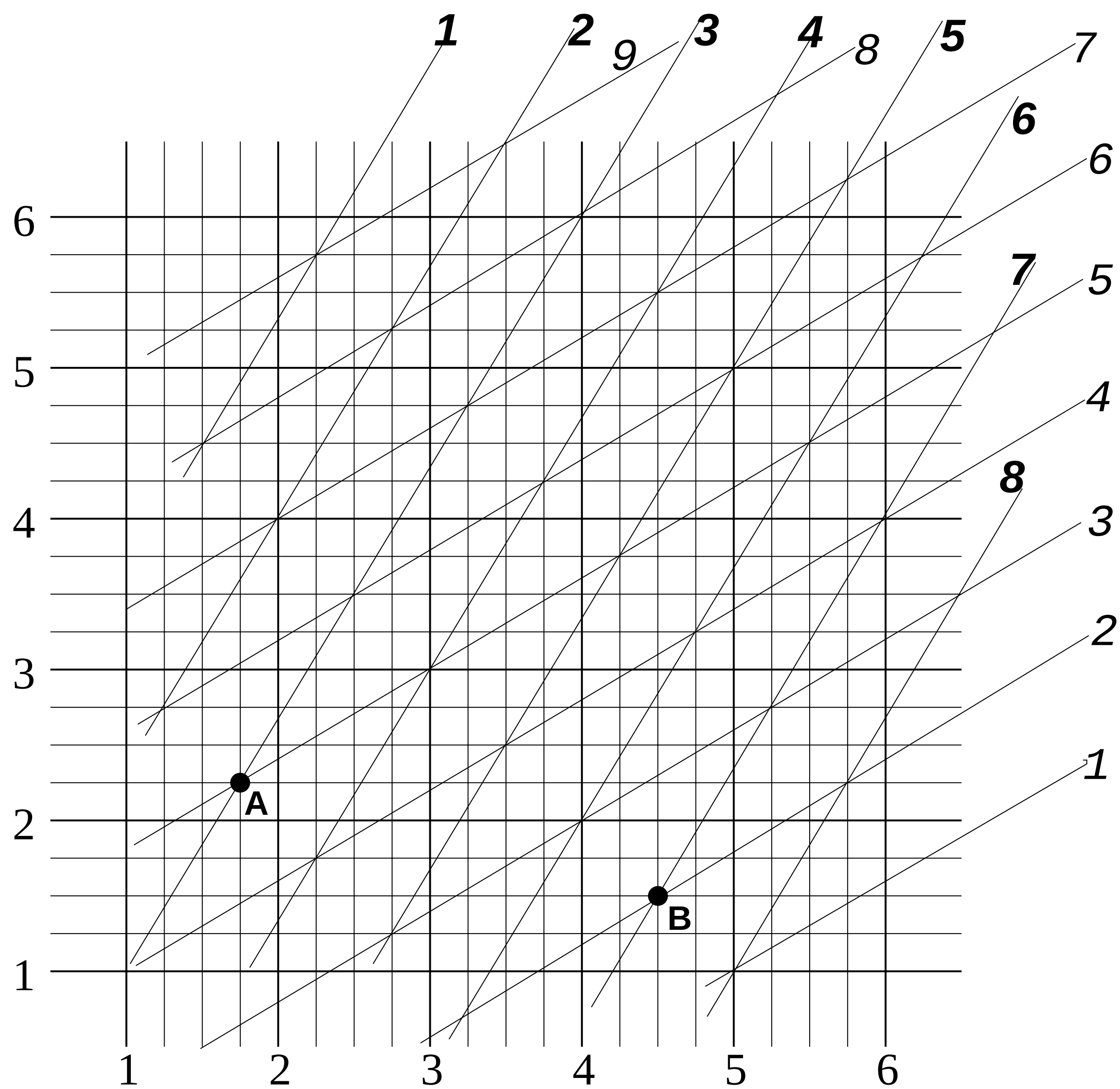

Figure 2.

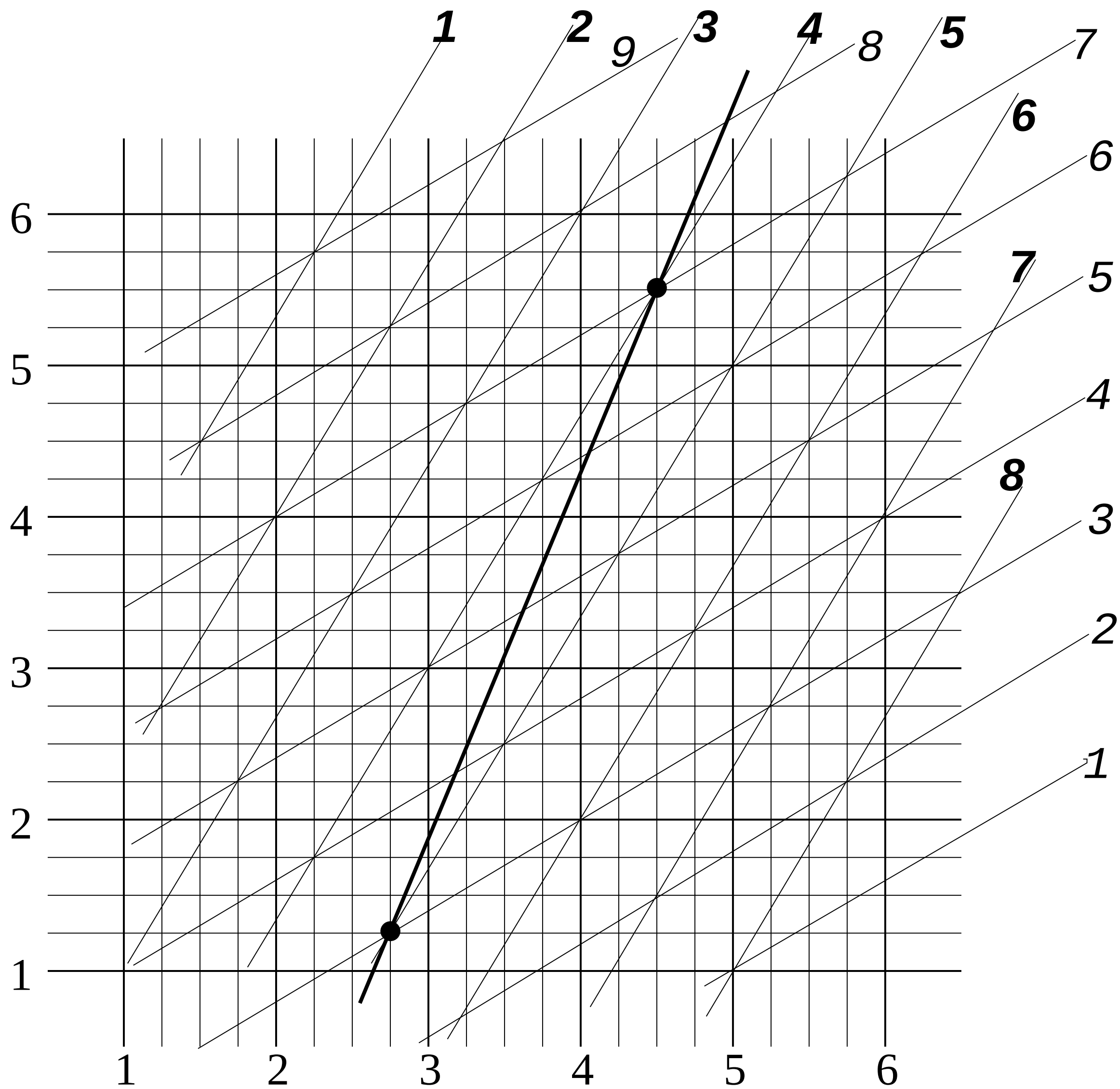

Figure 3.

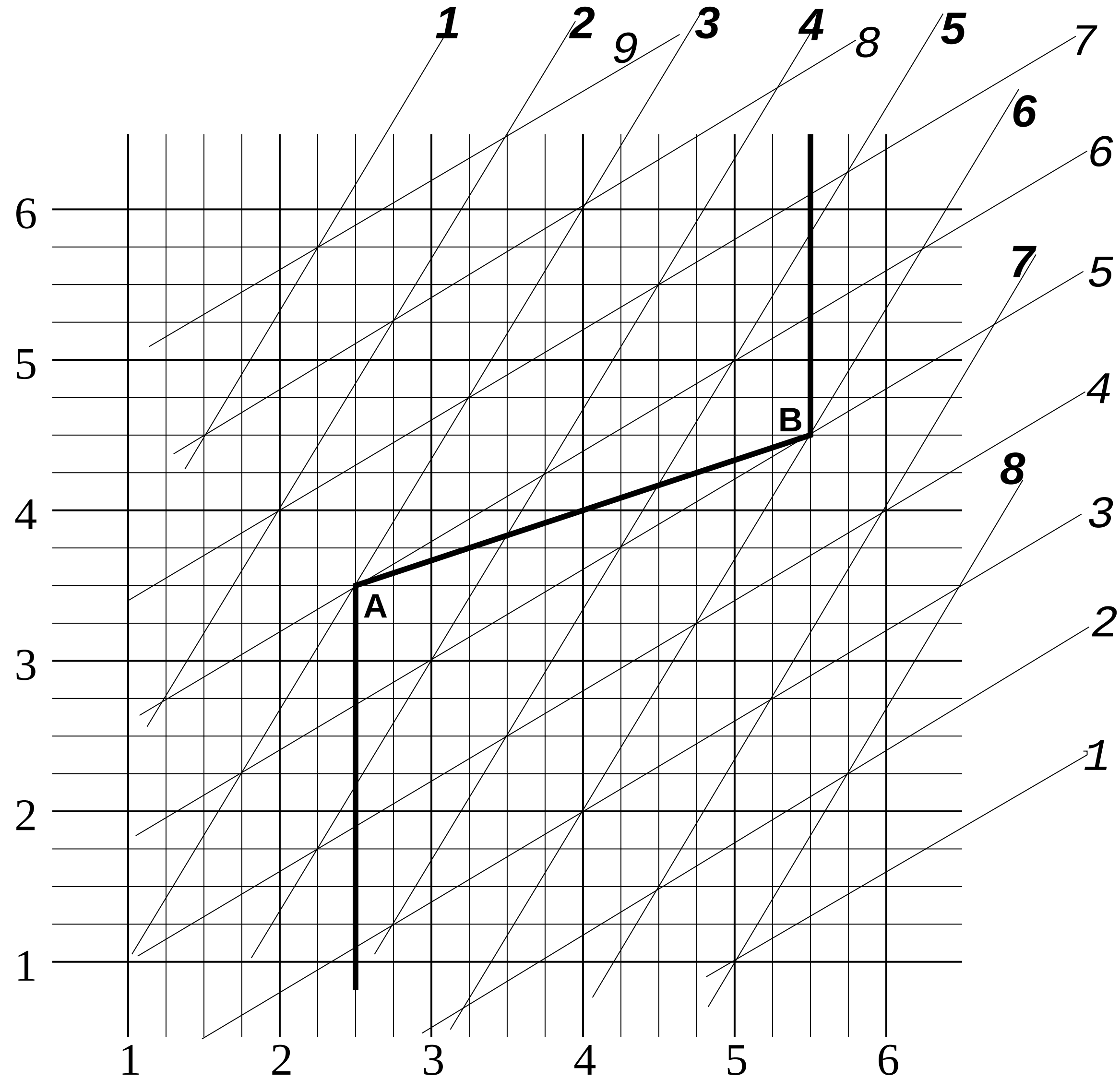

Figure 4.

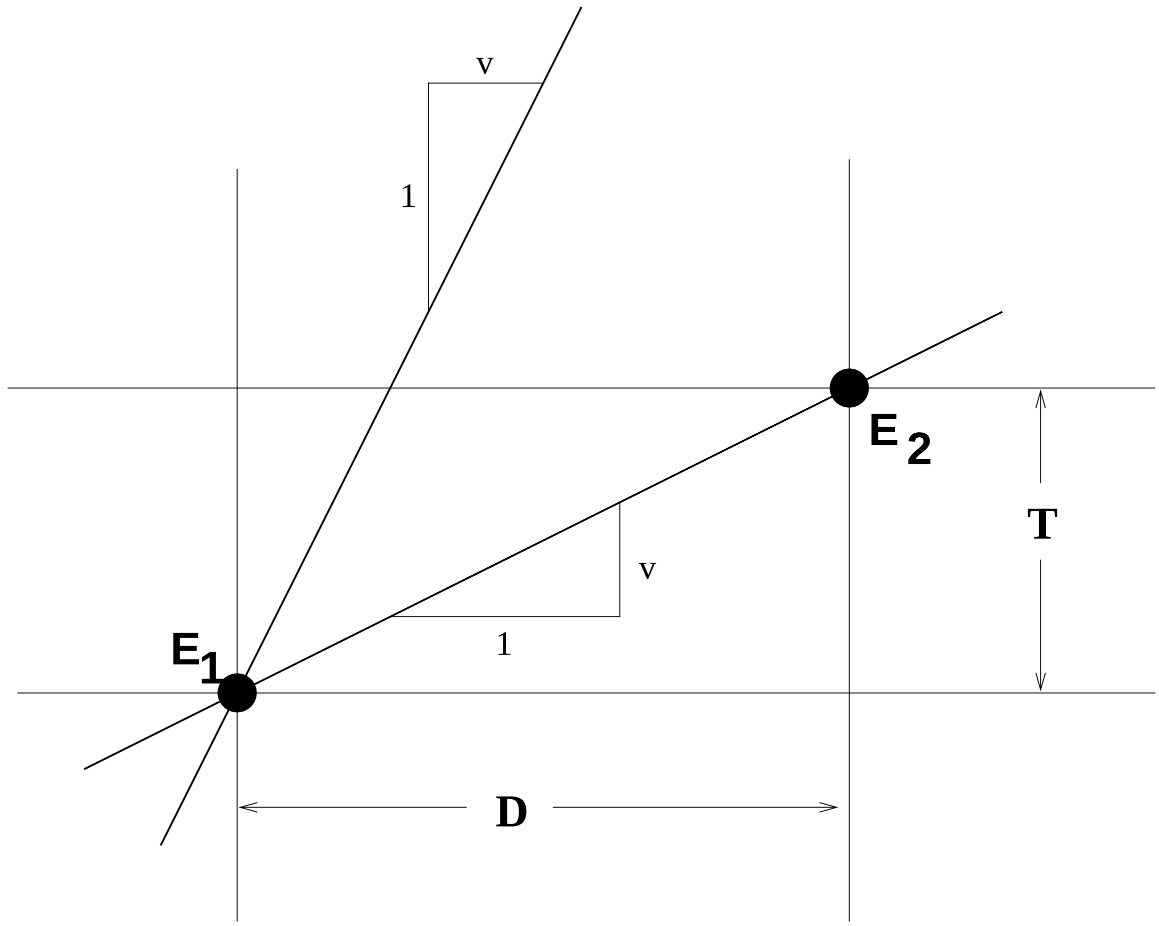

Figure 5.

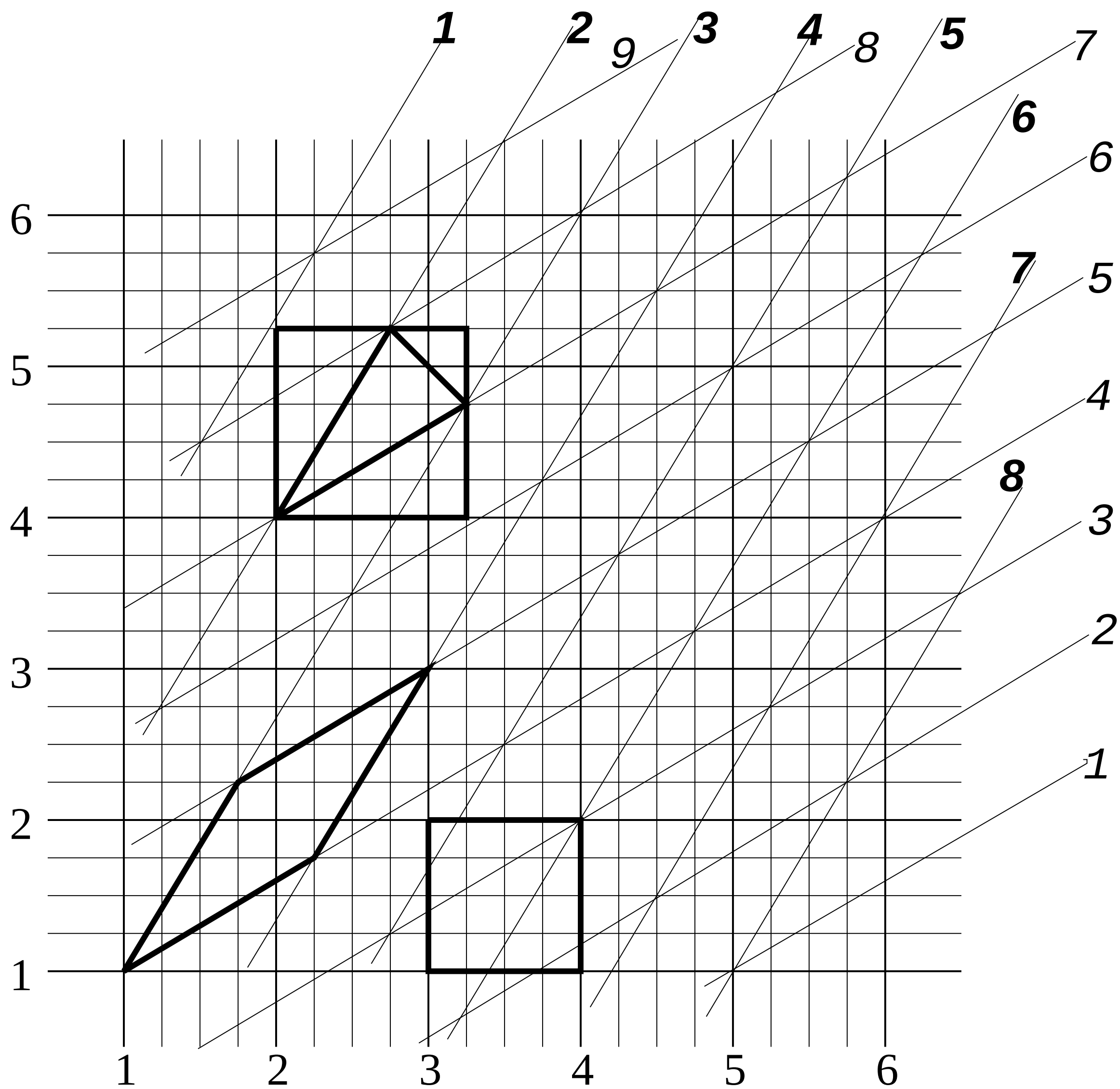

Figure 6.

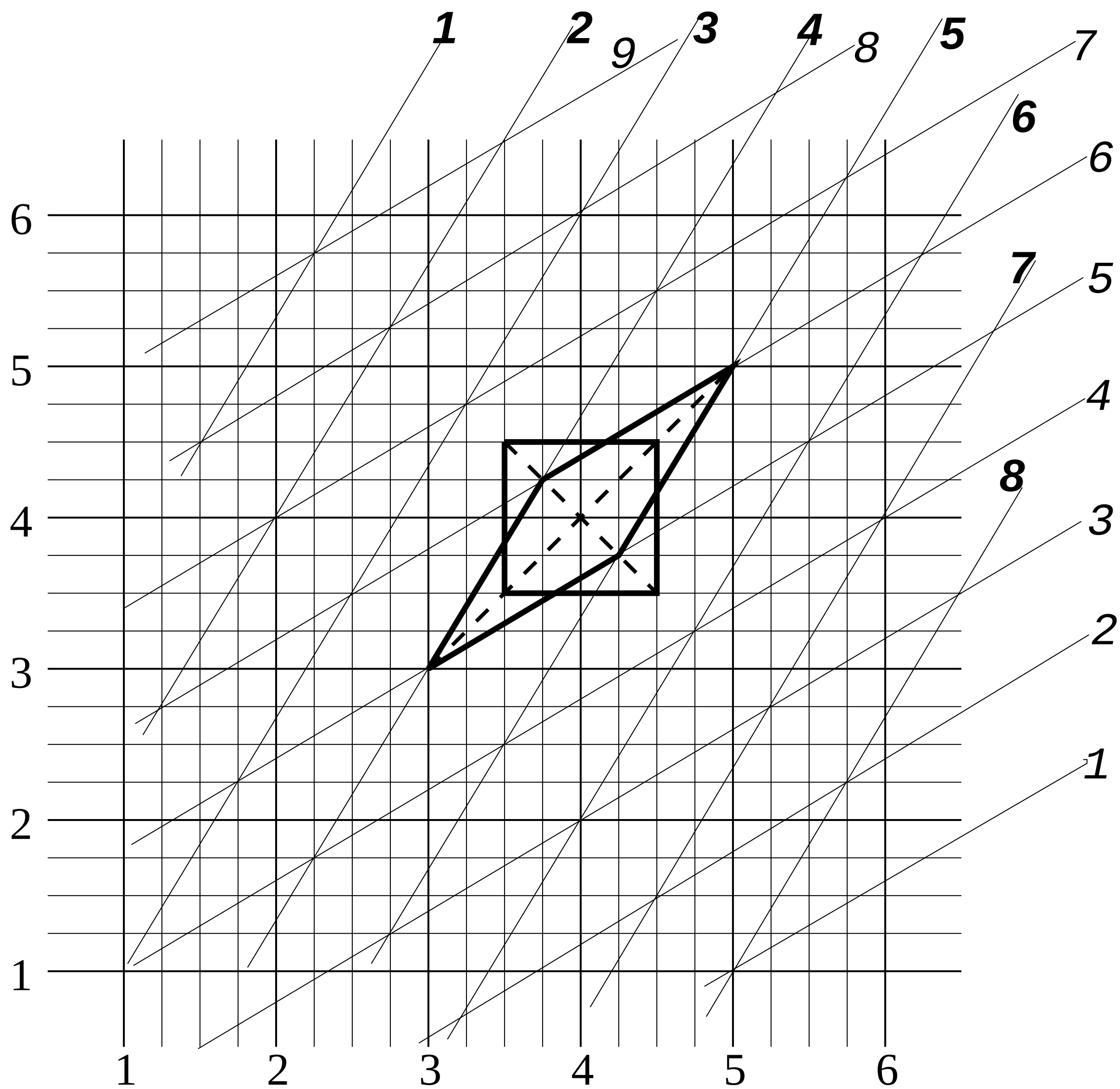

Figure 7.