

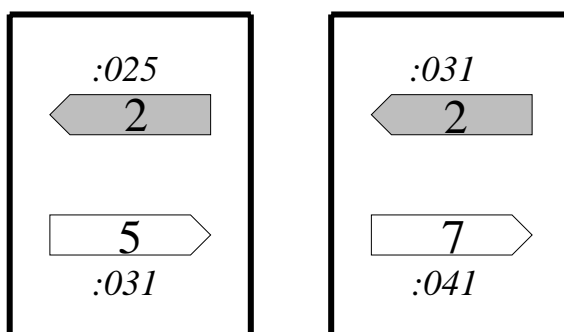
## Physics 209: Assignment #5

The questions that follow all refer to the Figures on page 4, which capture portions of three later stages in the evolution of the train of rockets described in Chapter 9 of the Lecture Notes, and which we will be examining in class. Except when otherwise noted, the questions are to be answer simply by looking at appropriate pairs of “photographs” of portions of the figures.

Your answer to each question should be in the form of a mini-figure showing two photographs of segments of the big figures, and a caption for the mini-figure that answers the question and explains how the answer follows from your mini-figure. Be sure you indicate which big figure each photograph is taken from. For example:

**Question:** *What is the velocity<sup>1</sup> of the grey train according to people on the white one?*

**Answer:**



*Here we see two different pictures in which the same grey rocket, #2, appears. In the picture from Fig. 1 it is opposite white rocket #5 at white time :031 ticks; in the picture from Fig. 2 it is opposite white rocket #7 at white time :041 ticks. Taken together the two pictures show that grey rocket #2 goes 2 white rockets ( $2 = 7 - 5$ ) to the left in a time of 10 white ticks ( $10 = 41 - 31$ ). Therefore the velocity of the grey train is 0.2 white rockets per white tick to the left. (This conclusion is independent of which grey rocket one happens to have picked.)*

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<sup>1</sup> When specifying a velocity you should give both the speed and the direction (right or left).

Your captions, like the above example, should consist of a few sentences that make it clear what the question was — it should not be necessary for a reader to have read the question to be able to understand the point being made by your figure and its caption — and how the figure answers the question.

1. What is the velocity of the white train according to people on the grey one?
2. By what factor do the white clocks slow down, according to people on the grey train?
3. By what factor do the grey rockets shrink, according to people on the white train?
4. By how much are the white clocks out of synchronization (in ticks per rocket) according to people on the grey train?
5. Show two photographs with the property that a single object, appearing between the trains in both photographs, would be assigned the same *velocity* by either train. What is that *velocity*? I stress “velocity” to contrast it with “speed”. Both trains will assign the same *speed* (1/8 of a rocket per tick) to an object that is stationary in the frame of reference of the page. But such an object is moving one way in the grey frame and the opposite way in the white frame. It therefore has different velocities in the two frames. An object moving at the invariant velocity has the same speed *and* the same direction of motion in both frames. I would suggest finding an answer by trial and error. Put the object anywhere in one of the figures, and hunt for a position in another one of the figures that works. *It is crucial for several of the questions that follow that you get the right value for the invariant velocity. If you find yourself having difficulty making the numbers come out right further along, come back to this question and check that you have really found the right answer.*
6. Show two photographs with the property that a single object, appearing between the trains in both photographs, would be assigned by both trains non-zero velocities that are not the same, with both speeds less than the invariant speed you found in 5.
7. [No mini-figure.] Confirm that the the two velocities you found in 6, the relative velocity of the two trains you found in 1, and the invariant speed you found in 5, are together consistent with the relativistic composition law for velocities

$$w = \frac{u + v}{1 + uv/c^2}.$$

8. Show two photographs with the property that people on the two trains would disagree which was taken first.
9. [Continue to use the mini-figure found in 8.] What would be the grey and white

velocities of an object (expressed as a fraction or multiple of the invariant speed you found in 5) that was present in both photographs?

10. Show two photographs with the property that an object appearing in both would be in two places at the same time, according to one of the trains. What would the velocity of such an object be according to the other train? Is this consistent with the relativistic composition law cited above?

11. By how much are the white or grey clocks out of synchronization (in ticks per rocket) according to people in the track frame (the frame in which the rockets are both moving with the same speed in opposite directions)?

12. [No mini-figure.] From your answers to 11 and 5 and from the  $T = Dv/c^2$  rule for clock asynchronization deduce the velocities of the white and grey trains with respect to the tracks.

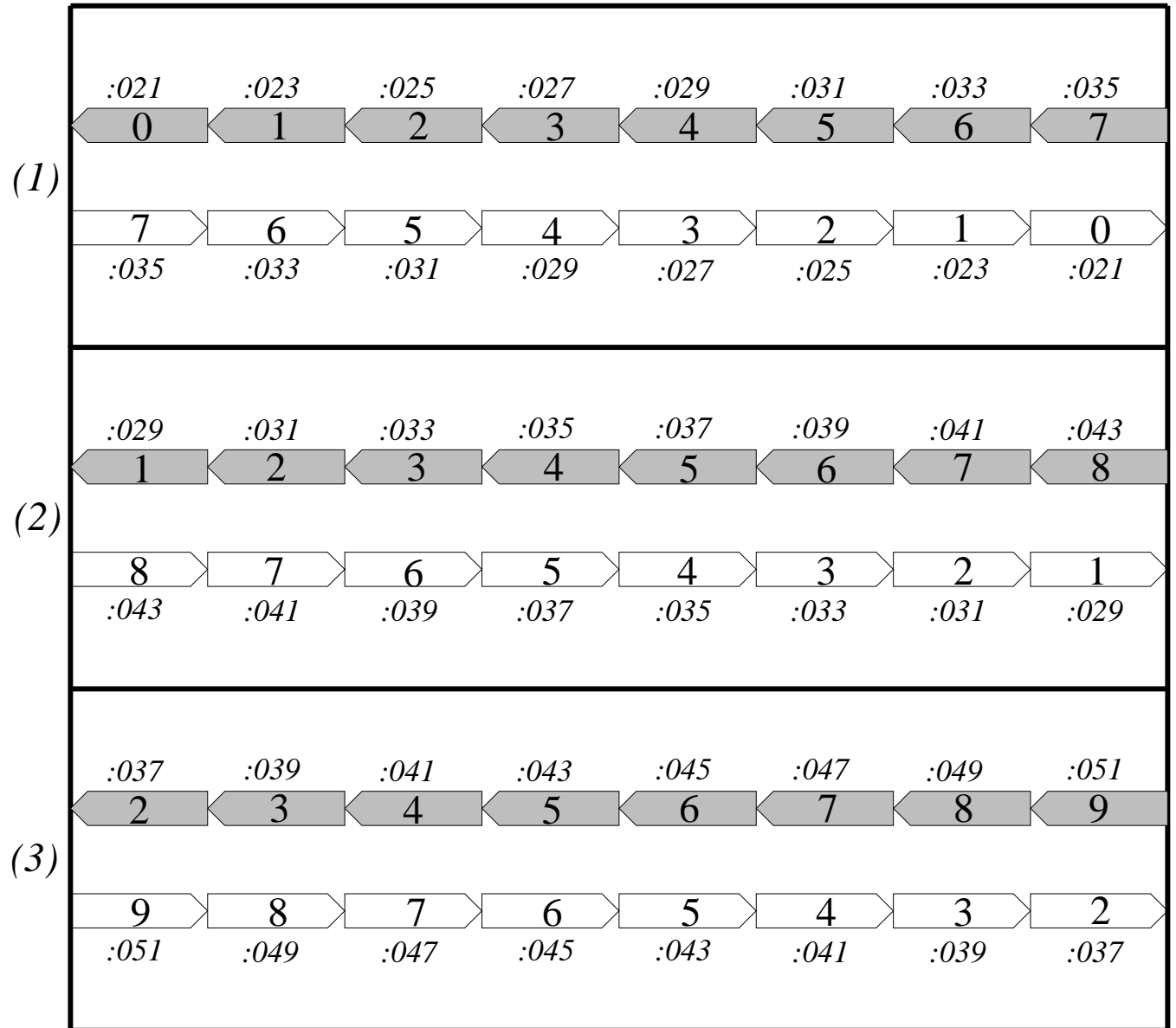
13. [No mini-figure.] Show that the two velocities you deduced in 12 do indeed combine under the relativistic velocity addition law to give the correct velocity of the grey train with respect to the white.

14. Show two photographs, one from Figure 1 and one from Figure 3, with the property that two events, one appearing in the first photograph and one appearing in the second, are time-like separated. Do the arithmetic necessary to get the value of the *interval*<sup>2</sup> between the events in both the grey frame and the white frame, and confirm that the two numbers are the same.

15. Do the same as in 14, but with two events that are space-like separated.

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<sup>2</sup> I remind you that the square of the interval between two events is  $|D^2 - c^2T^2|$  where  $D$  is the distance and  $T$ , the time, between the events. (It is enough to confirm that the squagrey intervals are the same — don't bother to take the square roots.) I also remind you that the two events are space-like separated if  $D^2 > c^2T^2$ , and time-like separated if it's the other way around,  $C^2T^2 > D^2$ .



Figures 1, 2, and 3.