Physics 209: Assignment #4

When we first discussed how to insure that two events in different places are simultaneous (and therefore how to synchronize two clocks in different places) we used a method that relied only on the constancy of the velocity of light. We were led in this way to the conclusion that whether or not two clocks not in the same place are synchronized can be a frame-dependent matter of convention. In particular clocks at opposite ends of a train of proper length D that are synchronized in the train frame are not synchronized in the track frame: according to the track frame the time indicated by the clock in front is behind the time indicated by the clock in the rear by Dv/c^2, where v is the speed of the train. Pursuing this line of investigation further, we were able to deduce that moving objects must shrink along the direction of their motion and that moving clocks must run slowly compared with stationary clocks, the shrinking and the slowing down factor s being given by

$$s = \sqrt{1 - v^2/c^2}. \tag{1}$$

Part I.

Now that we have all this information, we can examine in both the train and track frames yet another procedure for synchronizing two clocks in the train frame. Suppose the train is 1000 feet long and there is one clock at the rear of the train and another at the front. To synchronize the clocks, set the clock at the rear of the train to time 0, and at the same time[1] launch a signal that travels from the rear to the front of the train at 5/13 of the speed of light.[2] Since the speed of light is 1 foot per nanosecond (f/ns) it takes the signal 2600 ns (or 2.6 microseconds (μs)) to reach the front of the train. The instant the signal reaches the front of the train the clock at the front is set to read 2.6 μs. This is precisely what the clock in the rear reads at that moment, since that is how long it took the signal to get from the rear to the front. So the two clocks have been synchronized in the train frame.

[1] There is no ambiguity in the simultaneity of two things that happen at the same place.

[2] The important thing about this number is that it is not the speed of light: the signal need not be a light signal. I have given it this particular value because it makes the subsequent arithmetic quite simple, provided you keep the number as 5/13 and do not convert it into 0.384615....

Tell a story that describes this train-frame synchronization procedure in the frame of reference of a track along which the train moves at 3/5 the speed of light. You should freely make use of the fact that moving objects shrink and moving clocks run slowly by the appropriate value of the shrinking or slowing-down factor (1). You should be explicit about when and how you are using these facts.[3] You will also need to use the relativistic velocity composition law,

$$w = \frac{u+v}{1+\frac{u}{c}\frac{v}{c}}, \tag{2}$$

to find the speed at which the signal moves in the track frame.[4] Your story should be accompanied by illustrative figures, but it should not rely on the figures. The figures should make it easier for your reader to follow your line of argument, but a thoughtful reader ought to be able to follow the reasoning in your text without having to consult the figures at all.[5]

Your track–frame story should begin "A train of proper length 1000 feet moves along the tracks at a speed of 3/5 f/ns. Because of the Fitzgerald contraction its length is only. . . . When a clock attached to the rear of the train is set to 0, a signal is sent toward the front of the train at a (track frame) speed of. . . , " and continue in that vein. The main thing you have to figure out in your track-frame story is how long it takes the signal to reach the front of the train, but since you know how fast the signal is moving, how fast the rear of the train is moving, and how far the signal has to get ahead of the rear of the train to reach the front, this is elementary.[6] You have already been *told* what happens when the signal reaches the front—namely that the clock in front is set to 2.6μs, and since this is a statement about two events[7] at the same place *and* same time, track frame observers must agree that the clock in front is indeed set to 2.6μs when the signal reaches the front.

[3] You should not (and do not need to) use the Dv/c^2 rule quoted in the first paragraph above. The point of the exercise is to check at the end that the conclusion you reach agrees with the result you get directly from this rule.

[4] The reason I have taken the train-frame speed of the signal to be 5/13 f/ns is to make the resulting track-frame speed very simple; if you do not find something simple you have made a mistake in arithmetic. Do it again! All the arithmetic has been designed to lead only to simple numerical expressions. (In particular everything that ever appears under a square-root sign turns out always to be a perfect square.)

[5] This is how I try to use figures in the lecture notes: they can be very helpful, but they should not be essential. The text should be clear enough that people reading it with understanding can draw their own figures. You're just saving them the trouble of doing it.

[6] But be careful anyway!

[7] The two events are the signal arriving at the front of the train and the clock there being set to 2.6μs.

Since you have now figured out how long it took the signal to reach the front of the train and since you know the clock attached to the rear of the train read 0 when the signal left it,[8] you can also figure out what the clock attached to the rear of the train[9] reads at the moment the signal arrives at the front, according to people using the track frame. You therefore know whether the clocks end up synchronized in the track frame and, if not, what the discrepancy is in their readings.

End your story by checking that the discrepancy in the track frame is indeed exactly the one required by the Dv/c^2 rule stated in the first paragraph above.[10] I stress that the arithmetic analysis you need to do is very little and very simple. The real work of this Assignment is the writing: putting the numbers you get from that analysis together into a story that is clear, concise, and complete. (The story need not be very long, but it should be a model of clarity and precision.)

Part II (optional)

Do the same thing algebraically for the general case in which the train-frame speed of the signal, 5/13 f/ns, is replaced by a general speed u, the track-frame speed of the train, 3/5 f/ns, is replaced by a general speed v, and the train-frame length of the train, 1000 f, is replaced by a general length L. (As usual, do not attempt to write the essay until you have correctly done all the algebraic manipulations.)

[8] Another fact about two things happening in the same place at the same time.

[9] But remember that it is also moving in the track frame and therefore running slowly!

[10] If it is not, reexamine your story to find your mistake. Likely candidates for mistakes are forgetting that the track frame length of the train is less than its proper length, or forgetting that the clock sitting peacefully at the rear of the train is moving in the track frame and therefore running slowly in the track frame. Another source of error could be an incorrect use of the relativistic addition law to determine the speed of the signal in the track frame. Or perhaps you have done the track-frame analysis perfectly, but are not correctly applying the Dv/c^2 rule when you check your track-frame result to see if it agrees with the general rule.