Physics 209: Assignment #3

Part 5 of the lecture notes ("Simultaneity and Clock Synchronization") deduces directly from the constancy of the velocity of light that two events that are simultaneous in one frame of reference (the "train frame" used by Alice) need not be simultaneous in another frame (the "track frame" used by Bob). We shall call the precise quantitative rule for the time difference in the second frame "The Dv/c^2 Rule for Simultaneous Events":

If two events are simultaneous in the train frame, then in the track frame the event at the rear of the train occurs a time $T = Dv/c^2$ before the event at the front, where D is the track-frame distance between the events, v is the track-frame speed of the train, and c is, as usual, the speed of light.

The rule gives a simple relation between the track-frame time T between the events, the track-frame distance D between them, and the speed v of the track frame with respect to the frame (the train frame) in which the events are simultaneous. The rule is established in the lecture notes by analyzing, in the track frame, the procedure with light signals that is used in the train frame to confirm that the events are simultaneous. One might reasonably object to relying on the peculiar properties of light signals to establish anything so fundamental and surprising as the frame-dependent character of the simultaneity of a pair of events that happen in different places. But it turns out that any other sensible procedure to verify the simultaneity of two events leads to exactly the same conclusion. In this Assignment we shall investigate an alternative procedure.

Suppose we trigger the events at the two ends of the train with signals other than light, that both travel from the middle of the train to the ends with the same speed, which could be any speed at all. Since both signals travel with the same speed in the train frame and both have to cover the same distance to get from the middle of the train to one of the ends, the events at the two ends will be simultaneous in the train frame, whatever the (common) speed of the signals. It is highly convenient to take that speed to be the speed of light, as is done in the lecture notes, because the fact that light has the same speed in all frames of reference makes it quite easy to analyze the procedure in the track frame. But if we are willing to work a little harder we can use signals of any speed we wish, and reach exactly the same conclusion.

We have to work a little harder if the signals are not light signals, because of two complications:

1. In the lecture notes the speeds of both signals are c in both the train frame *and* the track frame. If the signals do not travel at the speed of light, we must use the relativistic law for the composition of velocities to deduce the signal speeds in the track frame from their values in the train frame. If a signal has velocity[1] u in the train frame and the train has velocity v in the track frame, then the velocity w of the signal in the track frame is given by:

$$w = \frac{u+v}{1+uv/c^2}. \tag{1}$$

2. The analysis in the lecture notes takes advantage of the fact that the speeds of the two signals in the track frame are the same (namely c) in both directions. But now, although the two signals have the same speed in the train frame, because that speed is not the speed of light they will not have the same speed in the track frame. As a result, although the argument remains similar to that in the lecture notes, you cannot slavishly follow that analysis step by step. In all three of the parts that follow, I would suggest modifying the approach in the lecture notes along these lines:[2]

(a) Get an expression for the time T_r it takes one of the signals to reach the rear of the train in terms of the unknown track-frame length L of the train, the track-frame speed of the train, and the track-frame speed of the signal. (You get the track frame speed of the signal from its train frame speed by using the relativistic velocity addition law.)

(b) Get an expression for the time T_f it takes the other signal to reach the front of the train in terms of L, and the track-frame speeds of the train and that signal.

(c) Get an expression for the track-frame distance D between the places on the track where the two signals arrive at their respective ends, in terms of L, the track-frame time T between those two arrivals, and the track-frame speed of the train.

(d) You want to find the relation between T and D. You could do this if you could express L in terms of T in (c). But you can get a relation between L and T from the fact that (a) and (b) give T_r and T_f in terms of L, and T is just $T_f - T_r$.

yLet's do this in two special cases:

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[1] Remember that the speed of an object is always a positive number (or zero if it is not moving). If the velocity of an object is equal to its speed when it is moving down the tracks, then it is the negative of its speed, when it is moving up the tracks.

[2] I've labeled all times and distances the same way they are labeled in the notes. Read quickly through these steps now, and then follow them carefully when you start trying to carry out the argument in parts I, II, and (if you feel like it) III.

I.

Suppose the speed of the signals in the train frame is half the speed of light ($\frac{1}{2}$ f/ns) and suppose that in the track frame the train is going to the right at a quarter the speed of light ($\frac{1}{4}$ f/ns). An advantage of using feet and nanoseconds is that the speed of light is $c = 1$ f/ns, as a result of which the relativistic velocity addition law (1) reduces simply to

$$w = \frac{u+v}{1+uv}, \tag{2}$$

where u is the velocity of a signal in the train frame, v is the velocity of the train in the track frame, and w is the velocity of that signal in the track frame, all in f/ns.[3]

(1) Write a clear explanation of why it follows from (1) that in the track frame one signal is going to the right at $\frac{2}{3}$ f/ns and the other is going to the left at $\frac{2}{7}$ f/ns. (What would the corresponding speeds be if you used the non-relativistic rule, $w = u + v$?)

(2) Give the track-frame analysis of this train-frame synchronization procedure, using the track-frame velocities of the signals given in (1) above.[4] If you do it correctly you will find that this analysis leads right back to our old conclusion that in the track frame the event in the rear of the train happened a time Dv/c^2 before the event in the front, where D is the track-frame distance between the places where the signals reached the two ends of the train and v is $\frac{1}{4}$ f/ns, the speed of the train.[5]

Because we are dealing with a particular numerical example rather than the general case, the algebra you have to do is fairly simple, but take great care to add together correctly the fractions that arise! It looks for a while as if you are getting into some quite cumbersome arithmetic, but at the end, if you have done the arithmetic right, everything miraculously reduces to something very simple.

(3) Go through the same analysis you undertook in (2) above, but replacing the track-frame speeds for the two signals with the ones you would get if you used the old familiar non-relativistic velocity addition law. Show that the result of the new analysis is the old familiar nonrelativistic result that the signals reach the two ends of the train simultaneously in the track frame too.

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[3] Use the convention that velocities are positive if the motion is to the right and negative if the motion is to the left.

[4] It might help to follow steps (a)-(d) above, though these are by no means the only way to proceed.

[5] Because we are using feet and nanoseconds, and because $v = \frac{1}{4}$ f/ns and $c = 1$ f/ns, the result of your analysis should just be $\frac{1}{4}D$: the time between the two events in nanoseconds is a quarter of the distance between them in feet.

WARNING: You should not attempt to do the analysis as you write the essay. It is almost essential to draw some figures for yourself, cross parts of them out, write down some relations, see if they lead you where you want to go, correct the inevitable arithmetical mistakes, cross out incorrect or useless steps, etc., before you start to write the essay. When you are ready to write the essay the many pieces of paper you have been scribbling on will be quite useful to you, but you should not submit them with the essay itself. Your essay should repeat only those parts of your scribblings and diagrams that turned out, in the end, to be pertinent to the discussion in that essay. It is also very important that you understand in detail the argument in the lecture notes, before starting to embark on the generalization of that argument I am asking you to undertake here.

II.

Suppose, on the other hand, that the speed of the signals in the train frame is $\frac{1}{4}$ f/ns and the speed of the train in the track frame is $\frac{1}{2}$ f/ns. Show, as you did in part I, that you still get the correct answer Dv/c^2 for the time between the two events in the track frame. Note that now, because the train is moving down the tracks faster than the signals are moving in the train frame, you have to think some about the direction the signals are moving in in the track frame, and you must take care to pick the velocities in (2) to have the right signs.

III (optional)[6]

The general case. Show that you get Dv/c^2 for the track-frame time between the two events for *any* velocity v of the train in the track frame and *any* common speed s for the two signals in the train frame. For concreteness take the case corresponding to Part I, where the train-frame speed of the signals is greater than the track-frame speed of the train. Conceptually the problem is identical to the one you solved in Part I, but some of the arithmetic is now replaced by algebra. Just as the arithmetic seemed to be getting horrible but became quite simple at the end, so too will the algebra collapse to practically nothing after threatening to grow into something rather alarming.

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[6] Occasionally I may declare part of an assignment to be optional. This means that while I think it constitutes a useful part of the learning process, I recognize that some (but not all) of you might find it excessively burdensome or time consuming. Some good work on optional parts of assignments can help in the final grade determination, if you are balanced between two grades. (Nobody will be penalized for not doing the optional parts.)