Physics 209: Assignment #2

Part I.

This is to give you a little more practice in using the skills you developed in Assignment #1. Once again, assume that all speeds are so small compared with the speed of light that complications associated with the possibly strange behavior of moving clocks and meter sticks are of no concern.

The technique for answering the questions is the same as in Part III of Assignment #1. You're told how two balls behave under certain conditions, and are asked to predict what will happen under a set of conditions that does not fit into the scheme you've been told about. To do this you must first find a new frame of reference in which the unfamiliar conditions do reduce to the ones you know how to deal with. Use what you know to specify what happens in the new frame after the collision. Then translate the answer you have just described in the new frame back into the language of the original frame.

Having figured all that out, write an essay explaining how you got the answer that is so lucid that any intelligent person who had never thought about these matters before would get the point.

(a) You have two balls, A and B, which when fired at each other with equal speeds, each bounce back in the direction they came from with 80% of their original speeds. Consider now an experiment in which ball B is stationary and ball A is fired at it with a speed of 10 fps. Write a short essay, illustrated with some explanatory figures, that deduces how fast each ball moves after the collision, by examining this experiment in a frame of reference in which the balls move at each other with the same speed.

(b) This is intended primarily to sharpen further your powers of abstraction. The way in which you exploit the principle of relativity to find a frame in which you know the answer is the same as in (a), but figuring out what that frame is and how the velocities change as you change frames now involves a little algebra, rather than just simple arithmetic.

Consider a situation exactly as in (a) except that *(1)* when the balls are fired at each other with the same speed, the speeds after the symmetric collision drop to some unspecified fraction f of their original values (f was 80% in Part I) and *(2)* the experiment consists of firing ball A at a stationary ball B with some unspecified speed u (which was

10 fps in Part I, and is now any speed that is tiny compared with the speed of light). **(1)** Reformulate the argument you gave in (a) so it now leads to formulae, in which f and u appear, for the speeds of the two balls after such a collision. **(2)** As a check that your formula is correct, it should give the answer you found in (a) when you substitute 4/5 (i.e. 80%) for f, and 10 fps for u. **(3)** Note any other cases (i.e. other specific values of f) that come to mind where the general formula gives a familiar answer. (I can think of two.)

Part II.

Let's return to question III.B of assignment #1, to see what happens when the speeds are *not* necessarily small compared with the speed of light. To remind you, we have a big ball (B) and a small ball (s). If the big ball is stationary and the small ball is fired directly at it, the small ball simply bounces back in the direction it came from with the same speed, and the big ball stays at rest. We again want to know what happens when the small ball is stationary and the big ball is fired directly at it, but this time let us take the speed of the big ball to be 90% of the speed of light.

When the speed of the big ball was small compared with the speed of light we were able to answer the question using the non–relativistic velocity addition law:

$$w = u + v, \tag{1}$$

which expresses the velocity w of a ball in the station frame in terms of the velocity u of the ball in the train frame and the velocity v of the train in the station frame. Indeed, the simple addition of velocities expressed by (1) is so intuitive that it is possible to answer questions relating velocities in different frames without even being explicitly aware of using it.[1] If u or v are not small compared with the speed of light c, however, the addition law (1) must be replaced by the so–called relativistic velocity addition law:[2]

$$w = \frac{u+v}{1+\frac{u}{c}\frac{v}{c}}. \tag{2}$$

[1] Chapter 2 of the lecture notes, "Nonrelativistic addition of velocities" does make explicit the ways in which the addition law (1) comes into play.

[2] Chapter 4 of the lecture notes, "Relativistic addition of velocities", explain why (1) has to be replaced by (2). In this assignment I am asking you simply to *assume* the validity of (2) and investigate how it modifies the conclusions you reached on Assignment #1. Note that in using (2) [just as in using (1)] one takes positive or negative u, v, or w to describe motion down or up the tracks, even though u and v appear in (2) in a more complicated way than they do in (1).

You can figure out what the balls will do when the big one is fired at the small one at 90% of the speed of light, by reexamining the analysis you carried out in III.B of Assignment #1, when all speeds were small compared with the speed of light, with a view to seeing where you made use of the non-relativistic addition law (1). To apply a similar analysis in the present case, when at least some of the speeds are not small compared with the speed of light, you must replace each such application of the non-relativistic law (1) by an appropriate use of the correct relativistic addition law (2).

(a) Following this procedure, deduce the speed of the initially stationary small ball after the big ball hits it at a speed $\frac{9}{10}$c.

(b) Redo what you did in (a) for the general case in which the initial speed of the big ball has some unspecified value u, to get a formula expressing the speed v of the small ball after it is hit by the big one, in terms of u and c. Estimate by how much your formula differs from the non-relativistic answer ($w = 2u$) when the big ball moves at the speed of sound (1000 fps).

As always, the results of all your investigations should be presented in the form of a lucid and coherent essay.