5. Simultaneity and Clock Synchronization

Newton: *"Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external. . . ."* This is simply wrong—an old prejudice.

Two events in the train frame that happen at the same place but at different times, happen at different places in the track frame. This is clearly correct and quite banal—something we are all used to.[1]

Two events in the train frame that happen at the same time but at different places, happen at different times in the track frame. This is shocking.[2] But it is an immediate consequence of the constancy of the velocity of light, as we shall now see. Note that it is obtained from the banal statement by simply interchanging the words "time" and "place". We will be encountering many other unexpected symmetries between time and space.

The puzzlement we feel at the fact that a given pulse of light has the same speed in both the track frame and the train frame can be traced to a deeply ingrained fundamental misconception. Until we learn otherwise (and prior to Einstein in 1905, nobody had learned otherwise) we all have an implicit belief that there is an absolute meaning to the simultaneity of two events that happen in different places, independent of the frame of reference in which the events are described. This assumption is so pervasive in our view of the world that it is built into the very language we speak, making it extremely difficult to reexamine the question of what it actually means to assert that two events in different places are simultaneous.

Before embarking on such a reexamination, I digress to comment on the meaning of the term "event", which plays a fundamental role in the relativistic description of the world. An *event* is something that happens at a definite place at a definite time. It is, if you like, the space-time generalization of the purely spatial geometric notion of *point*. Like the concept of a point, the concept of an event is an idealization. No object we can

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[1] Example. Train frame: "I sat still in my seat and read the paper for 30 minutes." Track frame: "The passenger started the paper in Boston and finished it in Providence."

[2] Read it again if you were not shocked; keep reading it until you are shocked.

actually get our hands on has the property of zero spatial extension that characterizes a geometric point, and no process we will be talking about has zero extension in both space and in time.[3]

Whether or not we wish to view something as an event depends on the spatial and temporal differences we are interested in discriminating between. If, for example, the relevant time scale is years and the relevant distance scale is hundreds of miles, then it makes sense to view the meeting of a class between 1:25 and 2:40 in room 115 of Rockefeller Hall on the Cornell University camups as an event.[4] But if the relevant scales are minutes and feet, it does not. World War II, on the other hand, is not an event on the scale of years and hundreds of miles, but it is on the scale of the age and size of the galaxy.

So a phenomenon can be viewed as constituting a single event in a given frame of reference, if its temporal and spatial extension in that frame are both small compared to all other times and distances of interest.

How can we check whether two different events, happening in different places, that are simultaneous in the train frame are also simultaneous in the track frame? To be concrete, suppose one event consists of making a mark on the tracks (as they speed past) from the rear of the train, and the other consists of doing the same thing from the front.[5] To begin with, how do people on the train persuade themselves that the two marks are made at the same time?

Well, one could provide both ends of the train with accurate clocks, and agree that each mark is made when the clock at its end of the train reads noon. But how can we be sure the two clocks are properly *synchronized?* How do you know they both read noon *at the same time*.

Evidently checking the simultaneity with clocks gets us nowhere, since confirming that the clocks are properly synchronized requires one to have precisely what we're trying to construct: a way to confirm that two events in different places—in this case, each clock reading noon—happen *at the same time.* This is a centrally important point. Two clocks in different places are useful only if they are properly synchronized. But "synchronized" means that the clocks have the same reading *at the same time*. Therefore you need a

[3] Zero extension in time means *instantaneous*. I know of no word (other than "point-like") that means zero extension in space.

[4] More accurately, it makes sense in a frame of reference tied to the surface of the earth.

[5] The two events could be anything else you like. (Bells being rung at the front and rear of the train, lightning bolts striking each end, etc.) But since it turns out to be useful to mark the spot along the tracks where each event occurs, it is convenient to simplify those two events to nothing more than those two acts of marking the tracks.

way to check that two events in different places are simultaneous, if you want to check that the two clocks in different places are synchronized. The question of whether clocks in different places are synchronized, and the question of whether events in different places are simultaneous, are simply different aspects of the same fundamental puzzle. You can answer one question if and only if you can answer the other.

Try again. One could bring the two clocks to the exact center of the train, directly confirm that they read the same when they're in the same place, and then carry them to the two ends. But how do you know that both clocks are running at the same rate as you carry them to the ends? Faced with a phenomenon as peculiar as the constancy of the velocity of light, it would be rash to assume that we knew anything about the rate at which a clock ran when it was moving along the train.[6] The straightforward way to check on whether the clocks have done anything peculiar while being carried to the two ends of the train, would be to compare what each read when it got there with the reading of a stationary clock at its end of the train. But we can only do this if those two stationary clocks are properly synchronized. This brings us right back to the original problem.

Ah, but suppose, even though we don't know how it might affect their rates, we moved the two clocks to the two ends in exactly the same way (except, of course, that one of the two clock-transportation procedures was executed in the opposite direction from the other.) Then however erratically its motion caused one clock to behave during the journey, the other, having experienced just the same kind of trip, would have run erratically in exactly the same way. So even if they lost or gained time because of their motion, the two clocks would still agree when they arrived at the ends of the train. That method of providing both ends of the train with synchronized clocks ought to work. And it does! In the train frame.

But now we are faced with another problem. Even if we did cleverly use two such synchronized clocks to confirm that two events at the two ends of the train were simultaneous in the train frame, observers in the track frame would not agree that the the two clock-transportation procedures were identical, because in the track frame motion toward the front of the train is *not* insignificantly different from motion toward the rear. For example the average speed at which each clock moves in the track frame is no longer the same (as it is in the train frame) for familiar reasons. Although people using the track frame would agree with somebody using the train frame that the reading of one clock, when it arrived at its end, was the same as the reading of the other, when it arrived at its own end,[7] they

[6] Later we will learn how to deal with this.

[7] This is because different frames can't disagree about things that happen *both* in the same place *and* at the same time.[8] In the present case the two events that coincide are (1) a clock arriving at the rear of the train and (2) that clock indicating a particular number.

would have to do a rather elaborate calculation to determine whether each clock reached its end of the train *at the same time* as the other clock. That calculation would have to figure out how fast each of the clocks was moving in the track frame, and how far it had to go. It could get quite complicated. It can, however, be done and it leads to a remarkable conclusion that we now extract by a much more straightforward stratagem.

The stratagem, like our earlier stratagem for finding the new velocity addition law, avoids all possible worries about misbehaving clocks by using a method to check that two events in different places are simultaneous in the train frame that makes no use of clocks at all. This method can be easily analyzed in the track frame too. It relies only on the fact that the speed of light is always c — one foot per nanosecond — regardless of the direction the light is moving and regardless of the frame of reference in which that speed is measured.

Why, you might ask, should we build such a strange fact into our procedure for determining whether two spatially separated events are simultaneous?

If you do ask, it is only because you have forgotten why we started worrying about whether simultaneity might depend on frame of reference. It was our hope that this might lead us to a clearer understanding of the constancy of the velocity of light. So what we are doing is perfectly sensible. We *start* from the strange fact of the constancy of the velocity of light, and see what it *forces* us to conclude about the simultaneity of events. We shall find that it forces us to conclude that the simultaneity of two events in different places does indeed depend on the frame of reference in a way that can be stated simply and precisely.

Note first that people on the train can exploit the fact that light travels with a definite speed c to arrange that the two marks on the tracks are made from the two ends of the train simultaneously. They place a lamp in the middle of the train and then turn on the lamp. Light from the lamp races toward both ends of the train at the same speed c. Since the light has to travel the same distance (half the length of the train) in either direction, and moves at the same speed in either direction, it arrives at the two ends of the train *at the same time.* So if the making of each mark on the tracks is triggered by the arrival of the light, the marks will certainly be made at the same time. We have thus managed to produce a pair of simultaneous events in different places without having to make any

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A similar pair of events coincide at the front of the train. Since track observers must agree with train observers on what each clock reads at the instant it reaches its end of the train, they must agree that the clocks read the same when they reach the ends. But they do not agree that the identical readings of the clocks means that it took an identical amount of time for each clock to get to its end, since the clocks were moving at different speeds in the track frame and therefore might be running at different rates. We shall see that identical clocks do indeed run at different rates in frames in which they move at different speeds.

problematic use of clocks.[9]

But how is this procedure interpreted in the frame of the tracks? People using the track frame will certainly agree that the lamp is indeed in the center of the train, for if the train is 100 cars long and the lamp is bolted down between cars 50 and 51, then there is no denying that it is indeed in the center.[10] But in the track frame when the lamp is turned on and the light starts to move toward the two ends, the rear of the train moves toward the place where the light originated and the front moves away. Since the speed of the light in either direction is c in the track frame — remember we are using this strange fact, that the speed of the light is one foot per nanosecond in the track frame even though it is also one foot per nanosecond in the train frame — in the track frame it will clearly take the light less time to reach the rear of the train, which is heading toward the light to meet it, than it will take the light to reach the front of the train, which is running away as the light pursues it.

So people using the track frame will conclude that the light reaches the rear of the train before it reaches the front, and therefore that the mark in the rear is made before the mark in the front. The very same evidence that convinces people using the train frame that the marks are made simultaneously, convinces people using the track frame that they are not. *Whether or not two events in different places happen at the same time has no absolute meaning—it depends on the frame of reference in which the events are described.*[11]

Note next that people using the train frame, for whom the marks are made simultaneously, could use the arrival of the light signals to synchronize clocks at the front and rear of the train. Since people in the track-frame maintain that the mark in the rear is made *before* the mark in the front, the track people would also maintain that the synchronization

[9] Notice that this procedure works for any two signals that move from the center of the train to the two ends, as long as they move at the same speed. If the common speed of the signals is not the speed of light, however, the track-frame analysis cannot be as concise as that given below, because the two signals have two different speeds in the track frame. Those two speeds can found with the help of the relativistic velocity addition law, and with a little more effort one can generalize to arbitrary signals the analysis given below for light signals. One finds, as one must, that this more general way to produce two simultaneous events in the train frame leads to the same relation between the track-frame time and track-frame distance between the two events, as we now extract with somewhat less effort using light signals.

[10] This is true even if the length of the train in the track frame is altered by its motion (as we shall soon see it is) because whatever that alteration might be, it would be exactly the same for both the front half and the rear half of the train.

[11] Notice that if you interchange time and space, that shocking assertion becomes quite humdrum: Whether or not two events at different times happen at the same place has no absolute meaning—it depends on the frame of reference in which the events are described.

procedure used by the train people had actually led to the clock in the front of the train being behind the clock in the rear.[12] *A disagreement about whether or not two events are simultaneous immediately implies a disagreement about whether or not two clocks are synchronized (and vice versa).*

It is not hard to make these disagreements quantitative. Let's analyze what has happened from the point of view of the track frame, where the train moves with speed v. It's convenient to call the length of the train L. I emphasize that by L we mean the length of the train *in the track frame.*[13]

In Part (1) of the figure[14] the light is turned on in the middle of the train and the two pulses of light (which we shall call photons) start moving from the center toward the front and the rear.

Part (2) of the figure shows things a time T_r later, just as the rearward moving photon encounters the rear of the train, which has been moving toward it. At the instant of encounter a mark is made at the place along the tracks where the encounter takes place. During the time T_r the photon (which moves with speed c) has covered a distance cT_r. But that distance is just half the length of the train, reduced by the distance the rear of the train (which is moving toward the photon with speed v) has moved toward the photon in the time T_r. So

$$cT_r = \tfrac{1}{2}L - vT_r. \tag{5.1}$$

Part (3) of the figure shows things a (longer) time T_f after the light was turned on in Part (1). At this moment the forward moving photon encounters the front of the train, which has been moving away from it. At the instant of encounter a mark is made at the place along the tracks where the encounter takes place. During the time T_f the photon (which moves with speed c) has covered a distance cT_f. But that distance is just half the length of the train, increased by the distance the front of the train (which is moving with speed v) has moved away from the photon in the time T_f. So

$$cT_f = \tfrac{1}{2}L + vT_f. \tag{5.2}$$

We want to find the time $T = T_f - T_r$ between the making of the two marks, so it is natural to subtract the second equation from the first, since the left side then becomes

[12] Make sure you understand this sentence before proceeding.

[13] Although we are used to thinking of the length of an object as being independent of the frame we measure it in, we can no longer take this for granted and, as noted earlier, we will indeed find it to be a false assumption.

[14] See the figure on page 9. It might be wise to stop at this point to examine the figure and read its caption, referring to the figure as you read, to make sure you understand both the figure and the caption.

$c(T_f - T_r)$ which is just cT. A second advantage of this procedure is that the unknown length L disappears from the result, which is simply

$$cT = v(T_f + T_r). \tag{5.3}$$

But what is $T_f + T_r$? Fortunately this quantity times c has a very simple meaning: $c(T_f + T_r)$ is just the sum of cT_r, the distance light travels along the track from the place where the lamp was turned on [shown in Part (1) of the figure] to the place on the track where it reaches the rear and the track is marked [shown in Part (2)]. And cT_f is the distance light travels in the other direction along the track from the place where the light was turned on [in Part (1)] to the place on the track where it reaches the front and the track is marked [shown in Part (3)]. Thus $c(T_f + T_r)$ is just the total distance D along the track between the two marks.

Replacing $(T_f + T_r)$ in (5.3) by D/c and dividing both sides of (5.3) by another factor of c so that T stands by itself on the left side, we have a relation between the track–frame time T between the making of the two marks and the track–frame distance D between them:

$$T = \frac{Dv}{c^2}. \tag{5.4}$$

We can abstract this into a general rule, by eliminating the talk of trains, tracks, and marks:

If two events[15] E_1 and E_2 are simultaneous in one frame of reference[16], then in a second frame of reference[17] that moves with speed v along the direction pointing from E_2 to E_1,[18] the event E_1 occurs a time Dv/c^2 earlier than the event E_2, where D is the distance between the two events in the second frame.

How big an effect is this? Suppose the two marks are 10,000 feet of track (about 2 miles) apart, and suppose the speed of the train is 100 f/s (about 70 miles per hour). Since the speed of light is a billion f/s, Dv/c^2 works out to $10,000 \times 100/(1,000,000,000)^2 =$ one trillionth of a second (one *picosecond*). The two events that are simultaneous in the train frame are a trillionth of a second apart in the track frame. Not the sort of thing you'd be likely to notice. On the other hand people who work with lasers these days are used to dealing with times a thousand times smaller than a picosecond (a *femtosecond*).

[15] E_1 is the marking of the tracks from the rear of the train and E_2 is the marking of the tracks at the front.

[16] The train frame.

[17] The track frame.

[18] From the point of view of the train frame the track frame is moving backwards with speed v.

Notice something else about the general rule. I've remarked above that if you interchange time and space, the surprising fact that two frames of reference can disagree about whether two events in different places are simultaneous, turns into the commonplace fact that two frames of reference can disagree about whether two events at different times happen in the same place. If we measure time in nanoseconds and distances in feet (or use any other units in which $c = 1$) then this intriguing symmetry under the interchange of time and space becomes quantitative as well as qualitative. The rule says the following:

> If two events are **simultaneous** in the train frame then in the track frame the **time** between them in **nanoseconds** is equal to the **distance** between them in **feet**, multiplied by the speed v of the train along the tracks (in feet per nanosecond).

Take that statement and interchange time and space in every word[19] that appears in boldface type, making no other changes. What you get is this:

> If two events are **in the same place** in the train frame then in the track frame the **distance** between them in **feet** is equal to the **time** between them in **nanoseconds**, multiplied by the speed v of the train along the tracks (in feet per nanosecond).

The second rule is nothing more than the precise quantitative formulation of the commonplace and familiar rule for how far something with a given speed goes in a given time.

In summary:

If two flashes of light travel from the middle of a train to the two ends, then in the train frame, of course, they arrive at the two ends simultaneously. But in the track frame a flash reaches the rear a time Dv/c^2 before a flash reaches the front, where D is the distance between the two places on the track where the flashes reach the front and rear, and v is the speed of the train.[20]

A final (*important*) remark:

If the times of the two markings are recorded in the track frame by two clocks, properly synchronized in the track frame and attached to the tracks at the places where the

[19] English lacks a single word that is to "simultaneous" as space is to time ("simullocated" is what's needed), so you have to replace it by the phrase "in the same place".

[20] It is important to emphasize that in the sentence beginning "But in the track frame..." the term "time" refers to track-frame time and the term "distance" refers to track-frame distance.

marks are made, how do people using the train frame, for whom the two marks are made simultaneously, account for the fact that the track-frame clocks read times that differ by Dv/c^2?

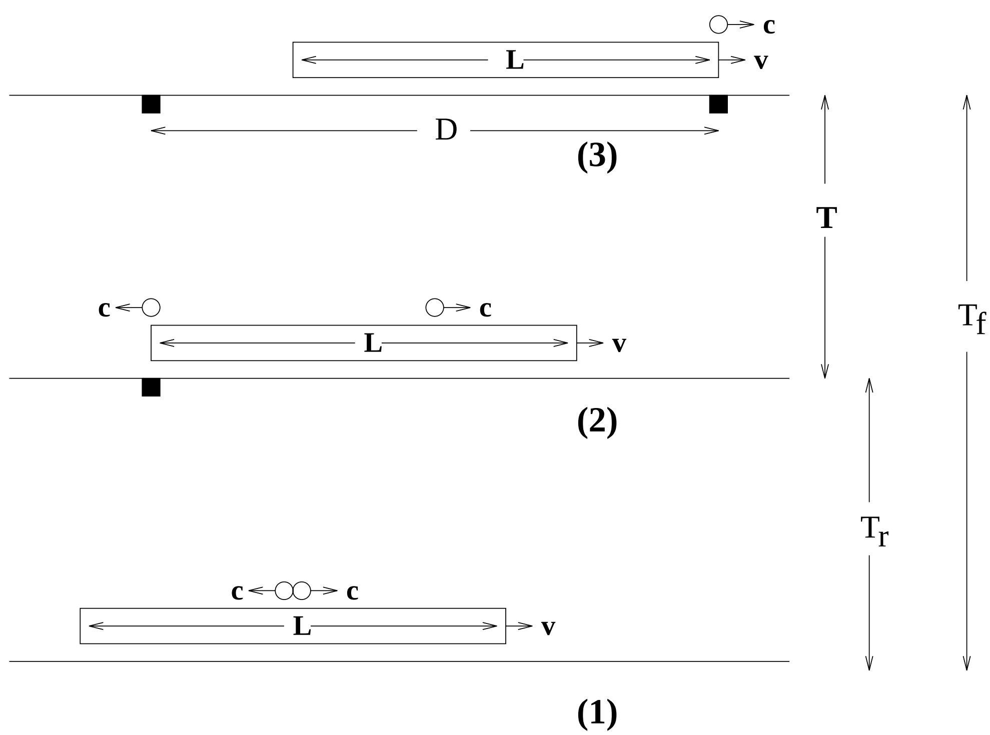
Easily! They say that the reason the track–frame clocks indicate the rear mark was made a time Dv/c^2 before the forward mark is that the track–frame clock that recorded the time of the rear mark is actually *behind* the track–frame clock that recorded the time of arrival of the forward mark by just that amount: Dv/c^2. This gives us the following rule:[21]

If two clocks are synchronized and separated by a distance D in their proper frame,[22] *then in a frame in which the clocks move along the line joining them with speed v, the clock in front is behind the clock in the rear by Dv/c^2.*

[21] This rule is the quantitative statement of what we had already noted qualitatively: that disagreements between frames about the simultaneity of events imply disagreements between frames about the synchronization of clocks.

[22] The *proper frame* of an object (or objects) is defined to be the frame of reference in which the object (or objects) are at rest.

The $T = Dv/c^2$ rule for simultaneous events



The figure depicts a series of events at three different times in the track frame. All lengths, times, and speeds shown in the figure are track-frame lengths, track-frame times, and track-frame speeds. For brevity I shall omit the phrase "track-frame" from each mention of a length or a time, but it is implicit throughout this caption.

The horizontal rectangle is a train of length L moving to the right with speed v. The white circles are photons that move with speed c. (1) Two photons originate in the center of the train, moving toward the front and the rear. (2) A time T_r after the first picture the photon on the left reaches the rear of the train. As it does so a spot (black square) is left upon the tracks to mark the place where the photon reached the rear of the train. (3) A time T_f after the first picture the photon on the right reaches the front of the train. As it does so a spot (black square) is left upon the tracks to mark the place where the photon reached the front of the train. This spot is a distance D away from the spot that was made in part (2). The time between the making of the two spots is $T = T_f - T_r$. As explained on pages 5 and 6 above, the relation between the time T between the making of the spots and the distance D between them is simply $T = Dv/c^2$.