Reply to "Neutral-fermion-soliton statistics in the short-range resonating-valence-bond state: A reevaluation"

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Haldane and Levine have questioned our Berry phase analysis [Phys. Rev. B **35**, 8865 (1987)] of the statistics of "spinon" and "holon" excitations in resonating-valence-bond states, claiming that the computations described in that work were performed incorrectly. We show below that their calculation does not disagree with ours, and in fact reproduces a limited subset of our results. We conclude with a brief caveat regarding the validity of the Berry phase approach to determining quasiparticle statistics.

In their Comment Haldane and Levine[1] examine our treatment[2] of the Berry phase calculation of neutral spinon defects in the short-range resonating-valence-bond model for the two-dimensional Mott insulator. As reported in Ref. 2, we obtained the following Berry phases: (a) For adiabatic paths involving a pair of spinons, the phase is π times the number of exchanges plus π times the area enclosed by the exchange path (in units of the area of an elementary plaquette). (b) For adiabatic paths involving a pair of holons, the phase is 2π per exchange. We interpreted these Berry phases as implying that spinons are fermions which move as if in an external magnetic field of one-half flux quantum per plaquette; holons behave as bosons without an external field. Note that Haldane and Levine only considered exchange paths which consist of a series of "quasiparticle hops" [see Fig. 1(a)]. It can be shown that such exchange paths necessarily enclose an odd number of plaquettes. Because of this, the phase of $2n\pi$ per spinon exchange that Haldane and Levine computed is consistent *either* with the conclusion that the spinon is a boson, *or* with our conclusion that it is a fermion in a background magnetic field of one half-flux quantum per plaquette. Thus, their results do not contradict our calculations. To obtain an unambiguous assignment of statistics, it is necessary to analyze exchange paths which enclose an arbitrary number of plaquettes, and hence one must consider "pair flips" [Fig. 1(b)] as well. (For several examples of exchange paths, see Fig. 2.)

Unfortunately, the above procedure for determining statistics is flawed because, as pointed out by Haldane and Levine and previously by us in Ref. 2 (although we did not at the time appreciate the significance of the observation), the exchange paths can easily be deformed in parameter space so that they enclose more or fewer "degeneracy

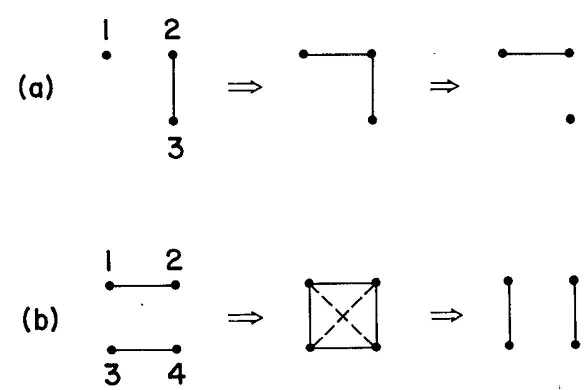

FIG. 1. "Elementary moves" used in the adiabatic transport of quasiparticles. The picture show the evolution of three- or four-site Hubbard Hamiltonians. Lines indicate links on which the hopping matrix is nonzero. (a) The "quasiparticle hop": Initially, the hopping matrix element between sites 1 and 2 is zero. It is adiabatically increased until it is equal to the hopping matrix element between sites 2 and 3. Then, the hopping matrix element between 2 and 3 is adiabatically decreased until it is equal to zero. At the end of this process the quasiparticle has moved from site 1 to site 3 or, equivalently, a valence bond has moved from link (23) to link (12). (b) The "pair flip": Initially, the only nonzero hopping matrix elements (shown as solid lines) are those between the pairs of sites 1 and 2 and 3 and 4, and have magnitude t. The hopping matrix elements between pairs 1 and 3 and 2 and 4 are adiabatically increased until they are equal to t, while at the same time the hopping matrix elements between 1 and 4 and 2 and 3 (dashed lines) are increased to t'. Finally, all the hopping matrix elements except those between sites 1 and 3 and 2 and 4 are adiabatically decreased to zero. At the end of this process, two horizontal valence bonds have been converted into two vertical valence bonds.

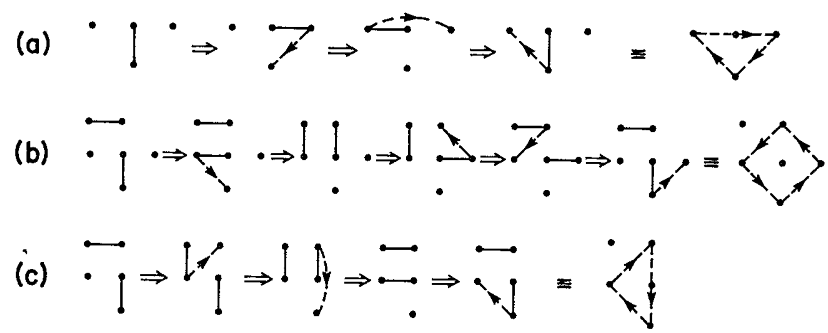

FIG. 2. Examples of paths which adiabatically evolve quasiparticles around closed paths. (a) A path involving one particle exchange and no pair flips. Notice that the path encloses an odd number of plaquettes (one). On paths involving no pair flips, the results of Haldane and Levine are in agreement with the results of Ref. 1. (b) A path involving one particle exchange and one pair flip. Notice that the path encloses an even number of plaquettes (two). Haldane and Levine did not analyze paths of this sort. (c) A path involving no exchanges and one pair flip. Notice that, in contrast to (b), this path encloses an odd number of plaquettes (one), so the parity of the enclosed area plus the number of exchanges plus the number of pair flips is the same in all three cases. Again, Haldane and Levine did not consider paths of this sort.

lines" (i.e., codimension-two manifolds in the parameter space of the Hamiltonian on which the ground state is degenerate). Thus, by deforming the path in small ways, the Berry phase can be changed by π. One would like to determine the manner in which each degeneracy line should be encircled by following adiabatic paths which correspond to low-energy states of the original Hamiltonian. We have, however, already butchered the original Hamiltonian so severely that it is unclear which paths are the most appropriate.

We conclude that Berry's statistical phase can only be used to resolve the issue of statistics if this phase is computed without recourse to approximation in a sufficiently simple model. Such a calculation was done in Ref. 3 for a holon in the quantum dimer model.[4] It was found that while the magnetic interactions favor a fermionic holon (in keeping with the results of Chakraborty and Read,[5] who rely on the Marshall sign to determine the appropriate paths around the degeneracy lines discussed above), the holon kinetic energy favors a bosonic holon. Since the holon kinetic energies are generally larger than the magnetic interactions, this analysis seemingly implies that a holon is more likely to be bosonic. (Of course, this argument presumes the validity of the approximations leading to the dimer model.) Finally, in Ref. 6, it is argued that the low-lying Bogoliubov quasiparticle excitations in a conventional superconductor can be thought of as neutral spin-$\frac{1}{2}$ quasiparticles; i.e., as the spinons in a resonating-valence-bond superconductor. These Bogoliubov quasiparticles are manifestly fermions.

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[5]B. Chakraborty and N. Read (unpublished).
[6]D. S. Rokhsar (unpublished).