

**Sethna *et al.* Reply:** Maritan *et al.* raise three issues of importance [1]. (A) They ask if it is possible that the phase transition we find in the shape of the hysteresis loop might have the same critical exponents as that of the equilibrium, *nonhysteretic*, random-field Ising model at its phase transition [their points (1), (2), and (4)]. (B) They introduce an “avalanche distribution” for the equilibrium model, which they suggest should obey the same scaling form and scaling relations we derive in our paper. (C) They propose a new scaling relation [their Eq. (2)] for the critical exponents of the avalanche distribution(s). We have of course considered these issues in some detail, and welcome the chance to discuss our tentative conclusions.

(A) *Same critical exponents?*—We had not noticed that the values of the critical exponents  $\beta$ ,  $\delta$ , and  $\nu$  were so similar to the corresponding equilibrium random-field Ising model:

Exponent	Hysteresis loop	Equilibrium RFIM [2]
$\beta$	$0.17 \pm 0.07$	$-0.1, 0.05$
$\beta\delta$	$2 \pm 0.3$	$1.6, 1.9 \pm 0.4$
$\nu$	$1 \pm 0.1$	$0.97, 1.29$

One must not confuse these two transitions. Unlike the equilibrium system, our system sticks in the first metastable state it finds (and thus is history dependent): our critical point is at a nonzero external field  $H_c(R_c)$ . Thus, although our mean-field equations are precisely the same as those for the equilibrium theory, we have two stable states below the critical randomness  $R_c$ , while the equilibrium theory selects the one parallel to the external field. We believe it is likely that the current rough agreement of the critical exponents is an accident: after all the two models have the same mean-field exponents and the same exponent relations.

(B) *Cluster flip distribution.*—At positive temperature, the equilibrium ground state of the random-field Ising model changes smoothly with external field [except at  $H = 0$  for  $T < T_c(R)$ ]. In particular, a cluster of  $N$  spins will “flip over” continuously over a range of fields  $\Delta H \leq kT/N$ . However, under a fixed  $\Delta H$ , large enough clusters should indeed act as if the temperature were zero, and an equilibrium cluster flip distribution under changes of external field makes sense. We agree that our scaling form and exponent relation for the athermal avalanche size distribution should also apply to this *equilibrium* cluster flip distribution [our colleagues’ Eqs. (1) and (3)]. However, there is no reason to believe that the two distributions are otherwise related. (Our theory may well apply to the finite temperature, nonequilibrium avalanches that one would find upon sweeping the field at low frequency and finite temperature.)

(C) *New exponent equality?*—Since the avalanche size distribution is associated with two new critical exponents  $\tau$  and  $\sigma$ , we need two exponent relations to derive the new exponents from the traditional ones. In our paper,

we reported only one. Maritan *et al.*, in their Eq. (2), propose a second exponent relation for the avalanche distribution exponents. This exponent relation is true for percolation, where it is called “hyperscaling.” In contrast to percolation this equation is not fulfilled by our mean-field exponents at the upper critical dimension of our system, which we found to be 6 from an expansion around mean-field theory [3]. To check our analytical results in high dimensions, we simulated the system in 3, 4, and 5 dimensions on the computer and found that the numerical exponents tend towards our mean-field results [4], rather than those obtained from a rival mean-field theory [5] as dimension 6 is approached. They also agree reasonably well (within the error bars) with our results from the  $\epsilon$  expansion about the upper critical dimension [3]. The proposed hyperscaling relation (2) does not seem to be correct in 4 and 5 dimensions either. It can be derived from the assumption that no more than one spanning avalanche can occur in a given system [6]. Initial numerical results seem to indicate that this assumption is wrong in our system above 3 dimensions: There, for systems near  $R_c$ , the number of spanning avalanches is growing with system size. For example, we found  $9.4 \pm 0.4$  spanning avalanches in systems of size  $20^4$  at near-critical randomness and  $10.8 \pm 0.5$  in systems of size  $40^4$ . In 5 dimensions there were  $32 \pm 2$  in systems of size  $10^5$  and  $51 \pm 2$  at size  $15^5$ . In 3 dimensions we have yet to simulate big enough systems to see clearly whether hyperscaling is violated there also. However, a relation involving dimension seems to be of little use if its realm of validity is confined to one single dimension.

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