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Hysteresis and avalanches: phase transitions and critical phenomena in driven disordered systems

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Abstract

We discuss Barkhausen noise in magnetic systems in terms of avalanches near a disorder-induced critical point, using the hysteretic zero-temperature random-field Ising model and recent variants. As the disorder is decreased, one finds a transition from smooth hysteresis loops to loops with a sharp jump in magnetization (corresponding to an infinite avalanche). In a large region near the transition point the model exhibits power-law distributions of noise (avalanches), universal behavior and a diverging length scale. Universal properties of this critical point are reported that were obtained using renormalization group methods and numerical simulations. Connections to other experimental systems such as athermal martensitic phase transitions (with and without 'bursts') and front propagation are also discussed. © 2001 Elsevier Science B.V. All rights reserved.

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Many physical systems that are far from thermal equilibrium show hysteresis in response to an external force or field ('the response lags the force'). A subset of these systems responds with collective or 'cracking' noise to a change in the driving force, just like wood snaps and crackles under an external load. A famous example is magnetic hysteresis accompanied by Barkhausen noise. In magnetic tapes, for example, the barriers to equilibration are so high, that on experimental time scales the system remains far from equilibrium, and as an external magnetic field is ramped up and down, the magnetization shows hysteretic response. The accompanying Barkhausen noise can be understood as the manifestation of 'avalanches' of reorientations of the magnetic moments, leading to magnetic domain wall motion and magnetic domain nucleation [1]. Similar behavior has been observed in ferroelastic materials, such as shape memory alloys or martensites [2]. A martensitic transformation is a diffusionless first-order phase transformation where the lattice distortion is mainly described by a homogeneous shear. Many metals and alloys with a BCC structure will upon cooling (or under strain) undergo this transition to a low-temperature close-packed structure. In athermal martensites, such as Cu–Zn–Al, thermal fluctuations do not play any relevant role — temperature acts as an external driving field. Similar to ferromagnets ramping magnetic field, the martensitic transition takes place as a sequence of avalanches in a broad temperature range. The analog of Barkhausen noise is the so-called acoustic emission due to elastic waves in the ultrasonic range, generated by propagating domain walls during the avalanches. Other examples of driven disordered systems showing hysteresis and avalanche response are superconducting vortices in the Bean state [3,4], fluids exiting from porous media [5–7], and even strike slip faults in the earth, where earthquakes can be considered as collective response to external stress on the fault [8].

One amazing feature that all these system have in common is that the observed distributions of avalanche

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sizes, times, energies, etc. are all very broad, in fact, following *a power law over several decades*. Moreover, the characteristic power laws for these various distributions seem to be *universal*, *i.e.* they are the same for entire classes of materials that have the same symmetries and dimensions. For Barkhausen noise, for example, roughly the same exponents have been obtained from experiments with a wide variety of materials [9–13]. The exponents found in Barkhausen noise measurements are different, however from, say, the corresponding exponents found acoustic emission measurements in Martensitic systems. Immediately several questions arise: (1) Why should there be broad distributions of avalanche sizes, times etc. rather than, for example, a Gaussian distribution with a well-defined mean avalanche size, time etc.? (2) Why does the same power law occur for a variety of different materials, independent of the underlying microscopic details? (3) Can we establish certain 'universality classes', which predict exactly how 'universal' these scaling exponents are, and explain why they are different for some systems? (4) Are there other universal quantities, except for these power-law exponents, that one can predict and measure?

Power-law scaling is a well-known feature in condensed matter physics. It is found, for example, at second-order phase transitions, at which systems become self-similar on all length scales. This self-similarity is reflected in the power-law scaling behavior of correlation functions and many other quantities at the critical point. Originally, phase transitions were observed in equilibrium systems, such as the liquid gas transition or the ferromagnetic paramagnetic phase transition in magnetic materials [14]. Non-equilibrium systems, too, can show phase transitions with associated universal power-law scaling behavior. Two main scenarios have been suggested to explain the observed universal power-law avalanche size distributions seen in so many experimental systems: either (1) there is an underlying phase transition, which implies that there is a *tunable parameter*, and only near the critical (phase transition) value of that parameter one finds power-law scaling behavior, or (2) the system is self-organized critical, i.e. one does not need to tune any parameter in order to find power-law scaling behavior on all length scales (up to the system size). Instead, special boundary conditions naturally force the system to operate near a critical point.

In this paper we address some of these issues using the non-equilibrium zero-temperature random-field Ising model and recent variants. The paper is organized as follows: we begin by introducing a general form of the model in paragraph a and then summarize the results in paragraphs b and c. The same paragraphs also contain initial comparisons with Barkhausen noise experiments quoted in the literature and specific suggestions for experiments suited to test some of the ideas presented here. Finally, paragraph d contains conclusions and out look.

(a) *The model*: Recently, hysteresis and avalanches in disordered magnetic materials have been modeled using several variants of the non-equilibrium, zero-temperature random-field Ising model (RFIM), which is one of the simplest models of magnetism, with applications far beyond magnetic systems (for a recent review, see Ref. [15] and also Refs. [11–13,16–18]). In contrast to some other hysteresis models, like the Preisach model [19] and the Stoner–Wohlfarth model [1], where interactions between the individual hysteretic units (grains) are not included and collective behavior in the form of avalanches is not an issue, in the RFIM the intergrain coupling is an essential feature and cause for hysteresis and avalanche effects. The zero-temperature nonequilibrium RFIM also shows the 'return point memory', or 'wiping out property', [17,18], which is a special (sub-loop-closure related) memory effect seen for example in martensites, charge density waves and many other dynamical systems, as well as the Preisach model. We have identified three sufficient conditions for the return point memory. (These conditions are indeed fulfilled by our model [17,18]). *The equilibrium* RFIM was originally introduced to study disordered magnetic materials in thermal equilibrium. We study the *nonequilibrium* version. To model is simply a caricature of the microscopic details in a magnet, near the critical point it correctly describes the long length scale behavior of systems with the same general properties such as symmetries, dimensions, interaction ranges and dynamics [11–13], as follows from renormalization group arguments.

In the RFIM, to each site i in a simple cubic lattice is assigned a variable s_i, a so-called 'spin', which can take two different values, $s_i = +1$ ('up') or $s_i = -1$ ('down').[2] (This corresponds to a real magnet where a crystal anisotropy prefers the magnetic moments or elementary domains that are represented by the spins to point along a certain easy axis.) Each spin interacts with its nearest neighbors on the lattice through a positive exchange interaction, J_{nn}, which favors parallel alignment. (For the behavior on long length scales, the exact range of the microscopic interaction is irrelevant, so long as it is finite.) Some variations of the RFIM also include *long-range* interactions due to the demagnetizing field and the dipole–dipole interactions. A general form of the Hamiltonian can be written as [15]

$$\mathscr{H} = -\sum_{\mathrm{nn}} J_{\mathrm{nn}} s_i s_j - \sum_i H s_i - \sum_i h_i s_i + \sum_i \frac{J_{\mathrm{inf}}}{N} s_i - \sum_{\{i,j\}} J_{\mathrm{dipole}} \frac{3\cos(\theta_{ij}) - 1}{r_{ij}^3} s_i s_j, \qquad (1)$$

[2] In this paper, we consider a lattice of classical spins. For an illustrative quantum mechanical description of magnetic moments in an external magnetic field see Ref. [20].

where H is the homogeneous external magnetic driving field, h_i is a local, uncorrelated random field, that models the disorder in the system, J_{inf} is the strength of an infinite range demagnetizing field, N is the total number of spins in the system, and J_{dipole} is the strength of the dipole–dipole interactions. The power laws are independent of the particular choice for the distribution $\rho(h_i)$ of random fields, for a large variety of distributions. Usually, a Gaussian distribution of random fields is used, with a standard deviation ('disorder') R. In a first approximation, the model is studied at zero temperature, far from equilibrium, to describe materials with sufficiently high barriers to equilibration, so that temperature fluctuations are negligible on experimental time scales. As the magnetic field is adiabatically slowly raised from $H = -\infty$ to $H = +\infty$ (or lowered from $H = +\infty$ to $H = -\infty$) two different *local* dynamics have been considered:

(1) in the first ('bulk') dynamics, each spin s_i flips when it decreases it own energy by doing so. We have studied this dynamics for the original RFIM without any long-range interactions, i.e. for $J_{\mathrm{inf}} = J_{\mathrm{dipole}} = 0$ [17,18,11–13]. This dynamics allows for domain nucleation (when a spin s_i surrounded by equal-valued spins flips in the opposite direction), and for domain wall motion, (when a spin flips on the surface of a preexisting cluster of uniform spins in a sea of opposite valued spins). A spin flip can trigger neighboring (or more generally, coupled) spins to flip as well, leading to an avalanche of spin flips, analogous to a real Barkhausen pulse. During an avalanche the external field is kept constant until the avalanche is finished, in accordance with the adiabatic limit in which we are interested. The model is completely deterministic – two successive sweeps through the hysteresis loop produce the exact same sequence of avalanches (since the temperature is set to zero). This dynamics may be appropriate to describe, for example, hard magnetic materials with strong anisotropies.

(2) The second dynamics is a 'front propagation dynamics' in which only the spins on the edge of an existing front (interface between up and down spins) flip if that decreases their energy. This dynamics can be used to model soft magnetic materials with a single or several noninteracting advancing domain walls and negligible new domain nucleation. The front propagation model without long range interactions ($J_{\mathrm{inf}} = J_{\mathrm{dipole}} = 0$) was originally introduced to model fluids invading porous media [21].

(b) *Results for bulk dynamics*: In this paper we mostly focus on the first dynamics where spins anywhere in the bulk are allowed to flip. In the following discussion, we have neglected the long-range interactions, setting $J_{\mathrm{inf}} = J_{\mathrm{dipole}} = 0$, which is appropriate for certain sample geometries [22,23]. We have simulated the model with up to a billion spins, averaging over many disorder realizations. The scaling analysis methods used to extract the critical exponents and an analytic renormalization group calculation to compute the exponents in $6 - \varepsilon$ dimensions are reported elsewhere [17,18,11–13]. Here we briefly summarize the results and then discuss suggestions for experiments to test these predictions.

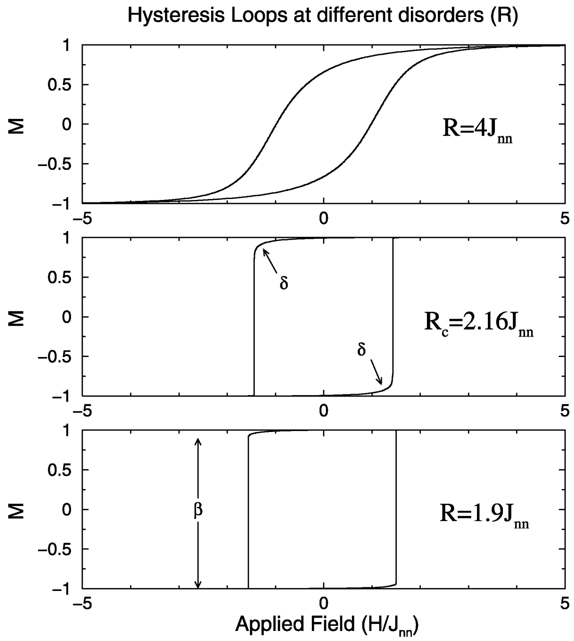

Fig. 1. Three hysteresis loops in our model, for systems at different amounts of disorder R. Each magnetization curve actually consists of many little steps, that are due to avalanches of spin flips. These avalanches are the analog of Barkhausen noise in real magnets. At disorders below a critical value $R_c = 2.16\,J_{nn}$ (in the simulations we actually set $J_{nn} = 1$), the hysteresis loops have a macroscopic jump in the magnetization M, which scales to zero as $\Delta M \propto (R - R_c)^{\beta}$. At R_c, the magnetization has a power law form $(H(M) - H_c) \propto (M - M_c)^{\delta}$.

Fig. 1 shows the associated hysteresis loops for different disorders R. Above a critical disorder $R_c = 2.16\,J_{nn}$ the hysteresis curve looks smooth, but it really consists of many little steps that are not resolved in the figure. Each of these steps corresponds to a finite avalanche of spin flips during which the magnetic field is kept fixed. Below the critical disorder the hysteresis loop has a macroscopic jump corresponding to an infinite avalanche sweeping through the system, thereby flipping a finite fraction of the system. The jump in the magnetization scales to zero as $\Delta M \sim (R - R_c)^{\beta}$, where $\beta \simeq 0.018$ is a universal prediction of the model for three-dimensional magnets [17,18]. At R_c the magnetization is described by a power law of the form $M(H) - M_c(H - H_c)^{1/\delta}$,

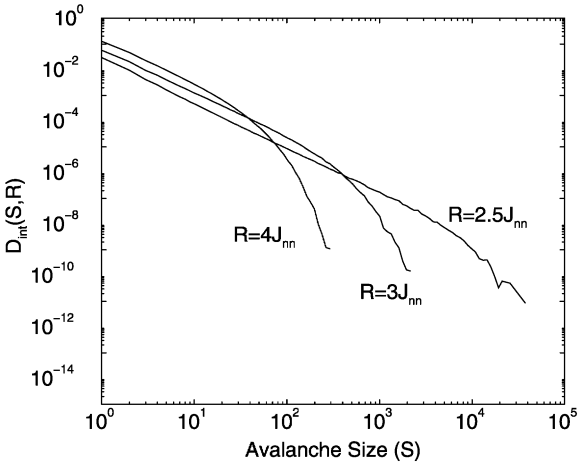

Fig. 2. Avalanche size distribution in the 3d zero temperature RFIM for several values of the disorder R for simulations with 320^3 spins, using the simulation code for the nonequilibrium RFIM that is available on the web. For a fun and instructive numerical simulation (with source code) of the non-equilibrium zero temperature RFIM, see http://www.lassp.cornell.edu/sethna/hysteresis/code. As the critical disorder $R_c = 2.16\,J_{nn}$ is approached the correlation length (and thus the cutoff to the power-law scaling of the distribution) diverges. Disorder averaged data of simulations with up to a billion spins and scaling collapses are presented in Refs. [17,18,24].

where δ is another universal prediction for experiments ($\beta\delta \simeq 1.8$ in three dimensions [17,18]), and $H_c = 1.435\,J_{nn}$ and $M_c = 0.19$ are *nonuniversal* constants. Note that the apparent dependence of the coercive field on the disorder is also *not* one of the universal predictions for experiments. Only the critical scaling exponents and certain scaling functions obtained near the critical disorder are expected to be universal predictions that can be compared to experiments.

Fig. 2 shows the distributions D_{int} (S, R) of avalanche sizes S obtained for hysteresis curves at several disorders above the critical disorder $R > R_c$. These curves correspond to the distributions of pulse areas in Barkhausen measurements (i.e. S is proportional to the integral of the voltage signal of the pulse over time) expected for samples with different amounts of quenched (structural or compositional) disorder. D_{int} (S, R) is proportional to a histogram of avalanche sizes observed during a sweep through the entire hysteresis curve. (Measurements over small windows of the magnetic field range lead to similar results, although with different (related) critical exponents, see [17,18]). At the critical disorder R_c the avalanche size distribution is described by a power law $D_{int}(S, R) \sim S^{-\tilde{\tau}}$ with the universal exponent $\tilde{\tau} \simeq 2.03$ in three dimensions. (In the literature the exponent $\tilde{\tau}$ is also referred to as $\tau + \sigma\beta\delta$ [17,18].)

Above the critical disorder R_c, the general scaling form is

$$D_{int}(S, R) \sim S^{-\tilde{\tau}}\mathscr{D}_{+}^{int}(S^{\sigma}(R - R_c)), \tag{2}$$

where $1/\sigma = 4.20$ is a universal exponent, and $\bar{\mathscr{D}}_{+}^{int}$ is a universal scaling function of the fitted form [17,18]

$$\bar{\mathscr{D}}_{+}^{int}(X) = e^{-0.789X^{1/\sigma}} \times (0.021 + 0.0002X + 0.531X^2 - 0.266X^3 + 0.261X^4). \tag{3}$$

As the critical disorder is approached from above, the avalanche size distribution D_{int} (S, R) shows more and more decades of self-similar (power-law) scaling behavior, up to an exponential cutoff size which grows as $S_{max} \sim (R - R_c)^{-1/\sigma}$, and reaches the system size near the critical point. The scaling behavior of the cutoff reflects the fact that there is a correlation length in the system that diverges at R_c, as is expected at a second-order phase transition [14]. Physically, this correlation length can be understood as the diameter of the largest avalanche of the power-law distribution. Interestingly, our numerical simulations indicate that the 'critical region' is remarkably large: almost three decades of power-law scaling in the avalanche size distribution remain when measured at a disorder R that is 40% away from the critical point. At 2% away, we extrapolate seven decades of scaling [17,18]. This may explain why in many experiments it does not even seem to be necessary to tune the disorder to see the critical power-law scaling over several decades: the used samples may just fall into this large critical region.

There are many related quantities that show similar scaling behavior near the critical point, for example avalanche durations, power spectra, various correlation functions, and the magnetization curve itself. The corresponding predictions for a set of critical exponents from Barkhausen noise experiments quoted in the literature are given in Refs. [24,15,11–13]. Generally, we found that the model predictions from simulations with up to a billion spins lie well within the error bars of the experimentally observed scaling exponents [17,18], although controlled experiments with tuned disorder to systematically test these ideas are still to be done. All Barkhausen noise experiments that quote power-law scaling exponents are typically done at a single value of the disorder R. Our model suggests that much more accurate information could be obtained from a *series of measurements for samples with different disorders* that allows for scaling collapses, and the comparison not only of critical exponents but also of universal scaling functions. Also, only a series of measurements at different disorders would be able to establish whether there is an underlying phase transition with disorder as the relevant tuning parameter at all.

Since the tuning parameter for the predicted disorder-induced phase transition is really R/J_{nn}, tuning the ferromagnetic interaction strength in experiments is expected to be equivalent to tuning the disorder (small coupling corresponds to large disorder and vice versa). In experiments, the disorder could possibly be tuned by annealing a sample with structural disorder at different annealing temperatures. In fact, initial measurements of $M(H)$ curves of a thin magnetic film whose structural disorder is reduced in steps through annealing at a number of different annealing temperatures, have lead to a sequence of hysteresis loops similar to Fig. 1 [22]. Moreover, experiments on Ni–Fe wires show that annealing the sample can indeed change the cutoff to the power-law scaling regime in the Barkhausen pulse area distribution [23], as suggested by our model. A quantitative scaling analysis along the lines presented here would be very interesting.

Introducing dislocations into the material by deformations may also correspond to tuning an effective disorder parameter. In that case, the effective disorder is probably correlated in space. Another way to change the disorder is by adding impurities to the sample, or by changing the grain size of the magnetic sample. It is known from nondestructive testing measurements that grain size indeed has an influence on the distribution of Barkhausen pulse times and areas. It would be interesting to see a systematic study of the effect of the grain size on the cutoff in the Barkhausen pulse area distribution. If avalanches can extend across grain boundaries and the individual grains do not act like single domains, one might expect the grain size to play a role similar to the disorder parameter R in our model. In martensitic systems a crossover from hysteresis loops with a jump or 'burst' to smooth hysteresis loops (ramping the temperature) has been observed by Olsen and Cohen [25], as they reduced the grain size of macroscopic, polycrystalline specimen of an Fe–Ni–C alloy. It seems clear that one should look for critical fluctuations near the crossover that would promote an interpretation of the grain size as the analog of the disorder parameter R.

Other experimental systems to which these ideas may apply, include superconducting vortex avalanches [3], the dynamics of ultrathin granular superconducting films in a parallel magnetic field [26], and fluid exiting in avalanches from porous media (specifically liquid Helium pumped out of Nuclepore) [5–7,27–29]. In some of these systems the symmetries and interaction ranges differ from our model, so that the values of the corresponding critical exponents might be different, but we still expect that the physics in these systems will be similar, with an analogous underlying non-equilibrium phase transition. Remarkably, our renormalization group studies show that the universality class of our critical point is actually very large: different kinds of disorder like random anisotropies, random bonds and random bonds with random fields are all expected to lead to the same critical scaling exponents [11–13,30] (see also Refs. [31][3] and [32][4]).

(c) *Results for front propagation dynamics*: Robbins, Cieplak, Ji, and Koiller originally introduced the RFIM without long-range forces ($J_{\rm inf} = J_{\rm dipole} = 0$) with the front propagation dynamics, to study fluids invading porous media, or more generally, the propagation of a *single* domain wall in disordered systems [21]. They found that below a critical disorder ($R < R_c^{\rm front}$) there is a second-order 'depinning' transition of the interface as the magnetic field is increased to a critical *field* and the advancing interface or domain wall is self-affine, i.e. it has no overhangs on long length scales. At the critical depinning field one finds power law distributions of avalanche sizes and times. The associated critical power-law exponents are different from the exponents of our RFIM that allows cluster nucleation anywhere in the system. At the critical disorder $R = R_c^{\rm front}$, one finds yet another set of power-law scaling exponents. Above the critical disorder ($R > R_c^{\rm front}$), the interface has overhangs and looks like the boundary of a percolation cluster.

At low disorders there are interesting connections between the self-affine propagating interface of Robbin's model and the infinite avalanche seen in the hysteresis model with bulk dynamics discussed in the last paragraph: the macroscopic jump in the hysteresis loop at low disorders, $R < R_c$, corresponds to an infinite avalanche sweeping through the system. We consequently expect that in the hysteresis model with bulk dynamics it would be possible to observe the interface critical exponents at the onset of the infinite avalanche in large enough systems. If the system is big enough, there will somewhere be a rare large cluster of flipped spins, even at relatively low magnetic fields. As the field is slowly raised, the surface of such a cluster is expected to act as a preexisting interface analogously to Ji and Robbins system. The small clusters that are flipped ahead of the interface in the hysteresis model with bulk dynamics are probably negligible on long length scales and are not expected to change the critical exponents associated with the interface progression. The onset field for the infinite avalanche in an infinite system should then correspond to the threshold field at which a preexisting interface gets depinned in Ji and Robbins system. Suggestions for

[3] Vives et al. [31] studied random-bond systems of sizes up to 40×40 and mixed random-field random-bond systems up to 100×100.

[4] Blossey et al. [32] studied the prewetting transition with our model using the efficient form of the brute-force algorithm described in Ref. [35], on systems up to 900×900.

experiments designed to test these ideas are given in Refs. [11–13].

In soft magnetic materials, long-range demagnetization fields are typically very important, and the Barkhausen noise is expected to be mostly due to the propagation of preexisting domain walls, without significant new domain nucleation. Interestingly, Urbach et al. [9,10] and Zapperi et al. [33] showed that at low disorders in the presence of *infinite range* demagnetization fields, the single domain wall (without overhangs) will via self-organization experience an effective driving field that is close to the depinning threshold, resulting in power law scaling behavior of the avalanche size distributions and other quantities ('SOC'). This is true even if in addition to the infinite range demagnetization fields, the magnetic dipole–dipole interactions are included in the model (although they change the critical exponents in three dimensions to mean-field exponents according to [33,34]. The associated (SOC) single interface critical exponents are again different from the ones associated with the disorder-induced critical point discussed in this paper.

(d) *Summary*: In summary, we have discussed the non-equilibrium RFIM with bulk dynamics as a model for hysteresis and avalanches in disordered systems. The model predicts a non-equilibrium phase transition from hysteresis loops with a macroscopic jump to smooth hysteresis loops as the disorder is tuned beyond a critical value. At the critical disorder the system has a diverging correlation length, and within a remarkably large surrounding critical region one finds associated universal power-law scaling of the noise (avalanches), the correlation functions, and the magnetization curve. While this model may be most appropriate for hard ferromagnets and rare earth materials, where strong local anisotropies prevent the formation of straight domain walls, in the low-disorder regime there are interesting connections to related single domain wall models with and without long-range demagnetization fields. It would certainly be most interesting to see results of controlled experiments on Barkhausen noise measured in a series of samples at different disorders. Similar disorder-induced critical points may also exist in other hysteretic systems with avalanches.

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