

Acoustic emission from crumpling paper

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(Received 7 December 1995)

From magnetic systems to the crust of the earth, many physical systems that exhibit a multiplicity of metastable states emit pulses with a broad power law distribution in energy. Digital audio recordings reveal that paper being crumpled, which can be easily held in hand, is such a system. Crumpling paper both using the traditional hand method and a cylindrical geometry uncovered a power law distribution of pulse energies spanning over two decades: $p(E) = E^\alpha$, $\alpha = 1.3 - 1.6$, with clearly nonexponential distribution over three decades. Crumpling initially flat sheets into a compact ball (strong crumpling), we found little or no evidence that the energy distribution varied systematically over time or the size of the sheet. When we applied repetitive small deformations (weak crumpling) to sheets which had been previously folded along a regular grid, we found no systematic dependence on the grid spacing. Our results suggest that the pulse energy depends only weakly on the size of paper regions responsible for sound production. [S1063-651X(96)04307-3]

PACS number(s): 64.60.Lx, 68.60.Bs, 46.30.-i

Everyone has had the experience of crumpling an unwanted sheet of paper into a compact ball prior to disposing of it. In one trial, after we hand crumpled a sheet of Xerox 4024 paper with an area of approximately 600 cm^2 and thickness 0.1 mm we produced a metastable object that appeared to be a roughly spherical ball with a volume of about 65 cm^3 . Although the crumpled object remains a sheet, a human observer with poor vision would perceive it to be a three dimensional object; it has been claimed that crumpled balls of paper and similar materials have a fractal structure with dimension $D = 2.3 - 2.5$ [1,2]. Although paper is one of the most ubiquitous composite materials, much is still unknown about the physics of paper. Recent research has addressed the tearing of paper [3-5] as well as the friction between two paper sheets [6]. Other researchers have recently investigated the crumpling sounds produced by the weak crumpling (small deformations) of a nearly flat but previously crumpled Mylar sheet [7].

Like a thin elastic sheet, paper tends to bend much more easily than it stretches. If one applies a slight stress to a paper sheet it will deform into a shape with zero Gaussian curvature almost everywhere, a developable surface [8]; the shape of a Möbius band, a paper strip with unusual boundary conditions, has recently been so modeled [9]. Unlike an elastic sheet, paper forms permanent creases under extreme local stress. When a crumpled ball is flattened out, a network of creases formed by crumpling is revealed. We refer to the polygonal regions of the sheet bounded by the creases as *facets*. Examining an unfolded crumpled sheet, one finds that the areas of the facets vary greatly and that a sheet with a crease network can be easily deformed into many different metastable states ranging from nearly flat to very compact balls.

Because paper makes an audible sound while being crumpled, we decided to probe the dynamics of crumpling by studying the sound produced. Crumpling sheets of several varieties of paper and similar materials such as plastic transparencies, we discovered that most of the acoustic power is emitted in the form of discrete pulses or acoustic emissions (AE).

Acoustic emissions are a versatile probe in science and engineering. Ultrasonic AEs provide insight into the dynamics of materials under both mechanical [10] and thermal [11] stress. AEs produced by the crust of the earth are an important probe for geologists; the largest and rarest AE events of the earth's crust radiate energy in excess of $4 \times 10^{12} \text{ J}$, endangering people and property on the surface of the earth. AE is particularly useful for the nondestructive testing of composite materials and has already been used to study the tearing of paper [3]; all 50 states now require AE safety inspection of fiberglass cherry picker arms [12]. AE has been used to study avalanches of glass beads [13]. The dynamics of magnetic systems produce (inaudible) Barkhausen noise, a pulsed magnetic signal with properties similar to AEs [14,15].

The folding states of a (nearly) inextensible sheet such as paper can be related to many other physical systems. Recently, connections have been drawn between the possible states of twinned martensites and possible foldings of a sheet [16,17]. In addition, "spin origami" mappings have been drawn between the minimum-energy states of the classical Heisenberg antiferromagnetic Kagóme lattice and foldings of an inextensible sheet [18,19]. Crumpling and folding transitions in equilibrium tethered membranes have also been a subject of recent interest in fields ranging from biophysics to superstring theory [20,21].

Crumpling paper produces pulsed AEs when facets suddenly buckle from one configuration to another; this can be verified by crumpling a sheet, uncrumpling it, and then slowly applying stress to the edges by hand. We observe that every discrete pop one hears can be traced to a single facet of the sheet undergoing a change of configuration; sounds do not appear to be produced directly by the formation of creases. Although it seems that several vibrational modes may be excited, both the oscillation frequency, on the order of a kilohertz, and the damping time, on the order of a millisecond, depend strongly on the type of paper but not on the energy of the pulse or the size of the sheet (see Fig. 1). Figure 2 is the complete acoustic record of one crumpling and Fig. 3 shows two individual pulses separated by our

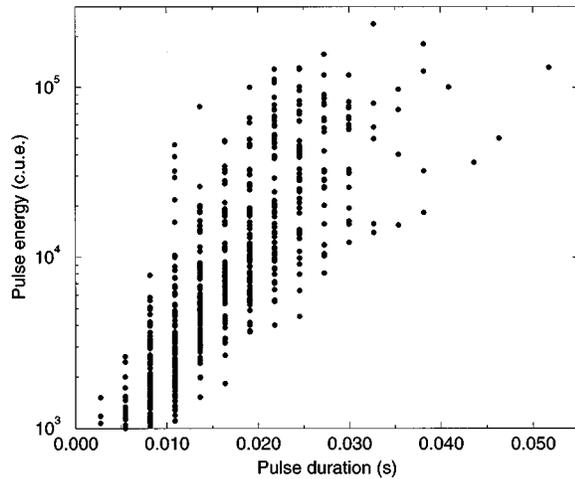


FIG. 1. Scatter plot of pulse duration versus pulse energy for cylindrical weak crumpling of Xerox 4024 paper with a 2-in. triangular grid. Horizontal axis is linear, vertical axis is logarithmic. Energy is measured in units of computer unit amplitude squared over time in seconds (computer unit energy or c.u.e.).

counting algorithm. Amplitude is measured in computer units (c.u.), 16-bit integers varying from -2^{15} to $2^{15}-1$ used to represent sound amplitudes in digital recording.

In our experiments we used three methods to crumple paper. In one, *hand crumpling*, the paper was crumpled by hand into a tight ball as slowly and evenly as possible over a duration varying from 63 s to 74 s. Initially it took us about 6 s to crumple a sheet in hand, but we found that it was essential to crumple very slowly for the computer to be able to identify individual events. Hand crumpling is interesting because it produces a very compact object, but it has the major disadvantage that it is imprecisely defined and irreproducible. Particularly, hand crumpling introduces an uncontrolled length scale related to the size of the crumpler's hands and fingers. For our other two methods, we fixed the paper to the ends of two hollow concentric cylinders, forming a cyl-

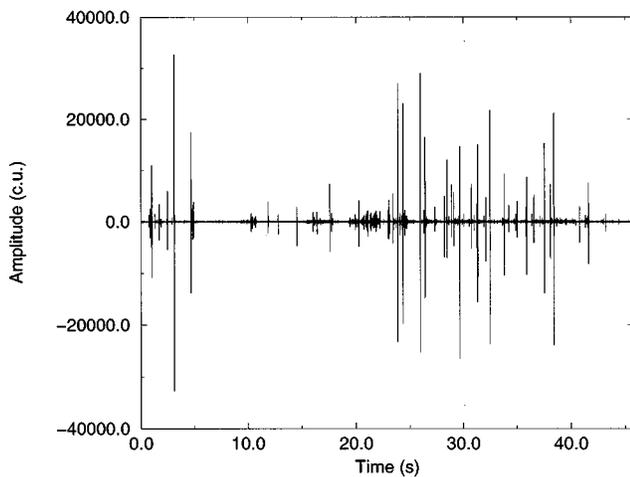


FIG. 2. Sound amplitude versus time: one entire strong cylindrical crumple, Xerox 4024 paper 8.5 in. square. Amplitude is measured in computer units (c.u.), signed 16-bit integers used to represent sound amplitude in digital recording.

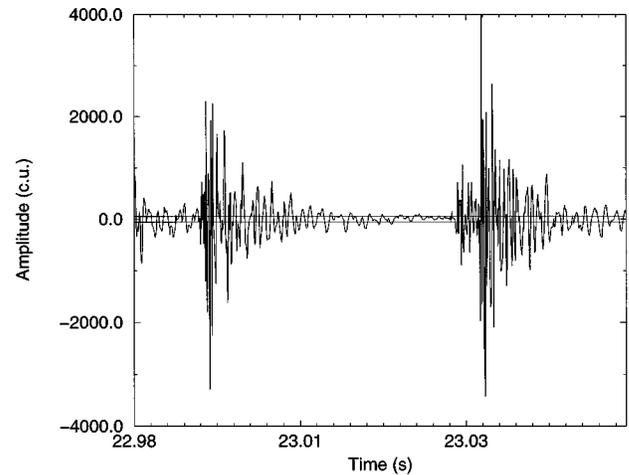


FIG. 3. Sound amplitude versus time: two adjacent pulses identified by our algorithm. The spacing of minor ticks is equal to the time bin duration in which energy was integrated, and the two superimposed lines show the threshold value. Bins were considered "active" when the energy inside equaled the bin length times the threshold amplitude squared.

inder of paper terminated by two metal cylinders. We then rotated the cylinders in opposite directions by hand. In all of our cylindrical experiments the paper sheet was a square with sides slightly shorter than the circumference of the cylinders, although other aspect ratios would have been possible. In the case of *strong cylindrical crumpling* we rotated the cylinders until it was impossible to rotate them further, producing a crumpled object not quite as compact as that produced by hand crumpling. We also performed *weak crumpling* [suggested by Eric Kramer (private communication): see [7]] experiments in the cylindrical geometry, rotating the cylinders only slightly back and forth — the range of rotation ending just before the free edges of the sheet were about to touch. Cylindrical crumpling has many advantages over hand crumpling: cylindrical crumpling can easily be performed slowly and can be scaled precisely in size. Because cylindrical crumpling can crumple a sheet by applying a well-defined strain to only the edges of the sheet, it can obviously be mechanized and may be easier to simulate and study theoretically. Weak crumpling, in addition, nearly eliminates noise from friction between paper surfaces and between the paper and the hands of the crumpler.

We recorded audio in an anechoic chamber using a Realistic 33-1090B Pressure Zone Microphone, and a Realistic 32-1100B preamplifier connected to a 486-based computer with a Turtle Beach Tahiti sound card. Sound was digitized at a sample rate of 11 000 samples per second in 16-bit linear pulse code modulation (PCM) for all of our pulse counting runs. Preamplifier and sound card gains were constant for all of our recordings, and all crumples were performed at a distance of 12 in. from the microphone.

Reference recordings taken at a sample rate of 44 000 samples per second and 16-bit linear PCM of crumpling demonstrated that the power spectrum for the crumpling of Xerox 4024 paper is peaked around 2 kHz, below the 5.5-kHz Nyquist frequency set by our usual sampling rate. Similar signals observed in magnetic [14,15] and martensitic [10] systems exhibit a broad power spectrum, due to either a

broad range of pulse durations and shapes [22] or on the time correlations between events [10]. Our pulses have much less structure. We find that large events are impulsive and the relationship between duration and energy is consistent with predominantly exponential decay, and we do not observe nontrivial scaling in the power spectrum (see Fig. 1).

To remove the dc offset from our data, we measured the median of the amplitude and subtracted it. We then integrated the energy in bins of fixed duration and compared the energy in each bin to a threshold. Contiguous runs of bins over threshold were considered to be single pulses and the pulse end time, duration, energy, and peak amplitude were written into a data file. We then plotted histograms using bins logarithmically spaced over pulse energy; error bars are $\pm 1\sigma$ assuming Poisson statistics (see Fig. 4). Figure 3 illustrates the process by which two pulses are identified. The rms amplitude of noise in the anechoic chamber with the human crumpler sitting motionless inside was 27.5 c.u.

Our pulse counting algorithm has two arbitrary parameters, the bin duration and the amplitude threshold. Because these parameters are arbitrary, one sign that we chose our parameters correctly is the insensitivity of our histogram to moderate changes in the parameters (of order 50%). When we chose a bin length much shorter than 1 ms, our oscillating signal drops below the threshold prematurely and our algorithm inappropriately fragments the pulses; in some of our early plots made before we started binning (equivalent to a bin of one sample) we observed false power laws spanning up to six decades in energy due to pulse splitting. In our early analysis, influenced by [15], we set our threshold to the median of bin power but we found with some data sets the histograms were strongly influenced by small changes in the threshold. Investigating this, we discovered that when our threshold was low, long (duration >50 ms) bursts of low amplitude noise caused presumably by paper friction or some mechanism other than that of interest were causing clearly separate events to be merged. We found that the severity of this would vary depending on the method and speed of crumpling, since slower crumpling would spread the pulses out in time, making them easier to separate and because some of the data sets, such as strong cylindrical crumpling of drawing paper, had much more unwanted paper noise than other sets, such as weak crumpling of paper with a grid.

We searched for a set of parameters that would accurately isolate pulses for all of our data sets and we converged on a bin length of 30 samples (2.7 ms) and a threshold amplitude of 50 c.u. (Threshold energy equals the threshold amplitude squared times the bin length.) We tested the pulse identification algorithm in two ways. (1) The output of the pulse counter was verified by comparing a sample of the pulses counted to a manual analysis of the set. Pulses identified by the algorithm were examined by eye to determine if they actually were impulsive events (in contrast to extended noise bursts) and to determine if they were inappropriately split or merged. We considered the output of the algorithm acceptable when 90% or more of the pulses in an energy bin were correctly identified. In addition, we checked the accuracy of integration for the weak crumpling sets (the sets of best quality) and it was found that our pulse counter with standard settings consistently underestimated the energy of pulses by

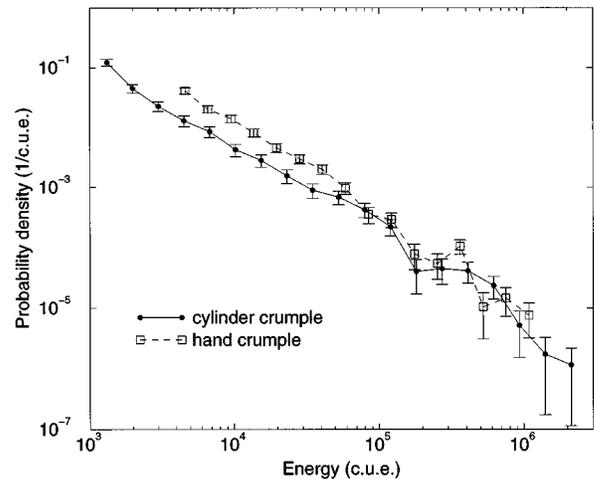


FIG. 4. Strong cylindrical crumples and hand crumples: Xerox 4024 duplicator paper was crumpled strongly in hand and using the cylindrical method. In both cases a sum of three crumpling runs is shown. Error bars are $\pm 1\sigma$ predicted by Poisson statistics.

730 ± 260 c.u.e. independent of pulse energy from smallest to largest. This is what we expect for our algorithm, since the threshold should cut off an exponential tail of roughly constant area. We estimated the cutoff energy below which identification errors were unacceptable for at least one set in each category. (2) We then developed a faster alternative test of pulse identification in which we would make pulse energy histograms increasing and decreasing the pulse threshold by 50%. Near the cutoff energy determined by the manual test the curve would secularly veer out of the error bars. We chose this as a criterion for setting the lower bounds on our histograms. One weak crumpling set (when we weak crumpled an initially flat sheet) had significant merging problems up to $E = 20\,000$ because the sheet was crumpled much more rapidly than later experiments. In our other weak crumpling sets, pulse identification was accurate down to $E = 1000$. In our strong crumpling sets we have problems with merging and spoofing below $E = 1000$ to $E = 10\,000$ depending on the set. We believe that with a lower threshold and shorter bin size we can accurately count pulses with lower energies in most of the weak crumpling recordings, but we chose to use a consistent set of parameters for all of our sets. Power law behavior appears to continue for another decade in weak-crumpling experiments of a triangular grid using less conservative parameters [26].

To search for time dependence in the energy distribution of sound pulses produced by strong crumpling we performed three crumples using the hand and cylinder methods with, respectively, letter size (8.5 x 11 in.) and 8.5-in. square Xerox 4024 paper. We subdivided the sets over time into thirds and combined the crumples to improve statistics. Figure 5 shows the result for cylindrical crumpling. About five exceptionally large events, spread out between the three crumples, cause the histogram for the first third in time to extend for a decade further than the others. There seems to be no systematic difference whatsoever between the last two-thirds of the crumpling process, or in the distribution of pulses of low to moderate energy. Crumpling by hand we found even less evidence for time variation; hand crumpling did not produce

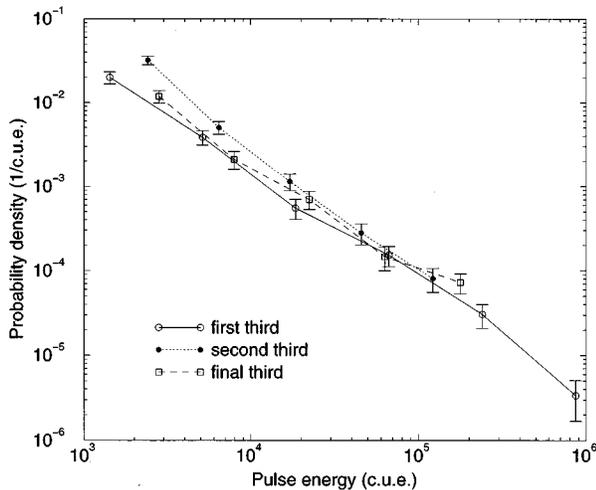


FIG. 5. Strong cylindrical crumples: time evolution. This is a sum of three runs performed with Xerox 4024 paper. The time series were divided in thirds over time. Although it appears that some very energetic events occur in the early stages of crumpling, there appears to be no systematic variation at other energies.

exceptionally large events in early crumpling nor any systematic variation in the pulse energy distribution.

To study finite size effects in paper crumpling, we performed sets of strong cylindrical crumples with square sheets of medium drawing paper (Carolina Pad Company item 54115) of sides 9 in., 6 in., and 3 in. and the cylinder diameter one-third the side of the paper. Drawing paper is considerably thicker than Xerox 4024 paper and presumably will have a longer characteristic length scale. A single sheet of 9 in. square paper was crumpled, as were four sheets of 6 in. \times 6 in. and nine sheets of 3 in. \times 3 in. The vertical axis of the histogram in Fig. 6 is normalized to sheet area. Since the sheet can only fragment into smaller facets with the passing of time, the natural assumption that pulse energy is deter-

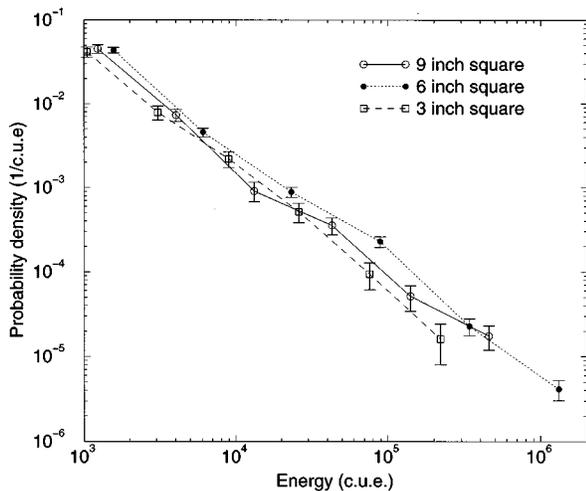


FIG. 6. Strong cylindrical crumpling: size variation. Medium weight drawing paper was cut into squares and crumpled using the cylindrical method. Larger numbers of smaller squares were crumpled to combat the loss of events, and the vertical axis is normalized over cumulative crumpled sheet area.

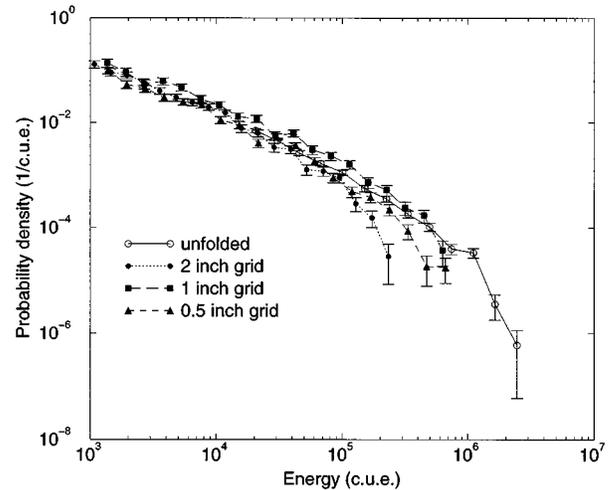


FIG. 7. Weak crumpling of triangularly gridded Xerox 4024 paper, normalized to 50 cycles. No systematic dependence of pulse energy distribution on grid scale is seen.

mined primarily by facet size is contradicted by the lack of both size and time dependence.

Because we were interested in isolating the effect of existing creases from that of self-avoidance, which would surely be important in a dense ball, we made recordings of the weak crumpling of precreased and crumpled sheets using Xerox 4024 paper on 3-in.-diam cylinders. These sets were of excellent quality, since pulses were well separated in time (≥ 100 ms), noise from paper friction was almost completely eliminated, and the number of pulses counted was much greater than the other experiments. We weak crumpled an uncreased sheet, a sheet of previously hand-crumpled paper, and a sheet of previously cylindrically crumpled paper. We also weak-crumpled sheets that had been hand creased along triangular grids with interline spacings of 2, 1.5, 1.0, 0.75, and 0.50 in. Figure 7 shows that the introduction of a creased grid clearly suppresses large events but shows no systematic relationship between the grid spacing and the energy scale at which suppression occurs. It proved possible to collapse the probability distributions for the various triangular grid spacings and the previously cylindrically crumpled grid by multiplying the energy and probability densities by constants, but the constants required appear to be random, showing no secular dependence on the grid size. Comparing early and late parts of weak-crumpling runs involving up to 100 cycles we found no evidence for time dependence.

Figure 8 compares weak cylindrical crumpling and strong cylindrical crumpling of an initially flat sheet. Since many other systems produce pulses with a power law distribution in energy [14,15,10] and it appears that the histograms could be well fit by a line on a log-log plot, we fit a power law of the form $p(E) = E^\alpha$ to our histograms. Over the energy range $E = 10^4 - 10^6$ we get $\alpha = -1.30 \pm 0.04$ for strong crumpling and $\alpha = -1.30 \pm 0.03$ for weak crumpling. We then combined all of the finite size runs using medium paper since we saw no dependence on size and fit an exponent of $\alpha = -1.32 \pm 0.03$ over the range $E = 10^3 - 5 \times 10^5$, which is compatible with the histogram from the 9-in. sheet alone with $\alpha = -1.24 \pm 0.06$. Larger events appear to be suppressed more strongly when a sheet is strongly crumpled by

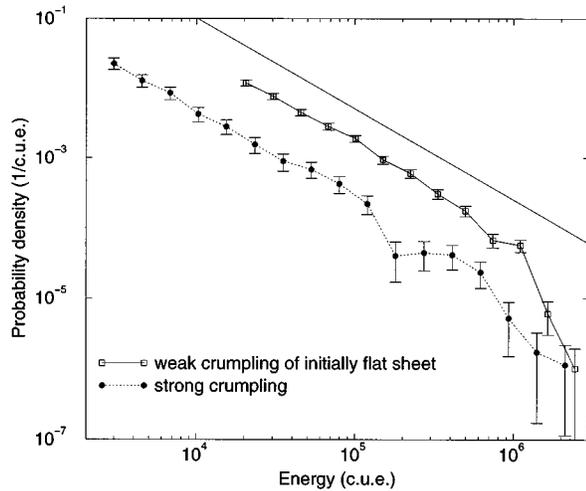


FIG. 8. Strong vs weak cylindrical crumpling: Xerox 4024 paper, sum of three runs of strong cylindrical crumpling is compared to 80 repetitive cycles of weak cylindrical crumpling. Overdrawn line has a slope of -1.3 .

hand ($\alpha = -1.59 \pm 0.09$ over the range of the plot) and when a previously hand-crumpled sheet is weakly crumpled on cylinders ($\alpha = -1.59 \pm 0.04$) (Fig. 9). We believe that the statistical errors in the fit exponents are much smaller than the systematic errors. The observed difference between strong hand crumpling of virgin paper and weak cylindrical crumpling of precrumpled paper is statistically significant (Fig. 9).

Our data are compatible with the assertion that the energy released when a facet buckles is insensitive to the size of the facet. Although it is possible that we are not probing a small enough length, we see no systematic dependence on facet size when we introduce a grid. In addition, since facets are formed by the fragmentation of larger facets, the size scale of facets on the sheet can only decrease over time; the lack of time dependence suggests a lack of size dependence. If we presume that a nearly constant fraction of the elastic energy

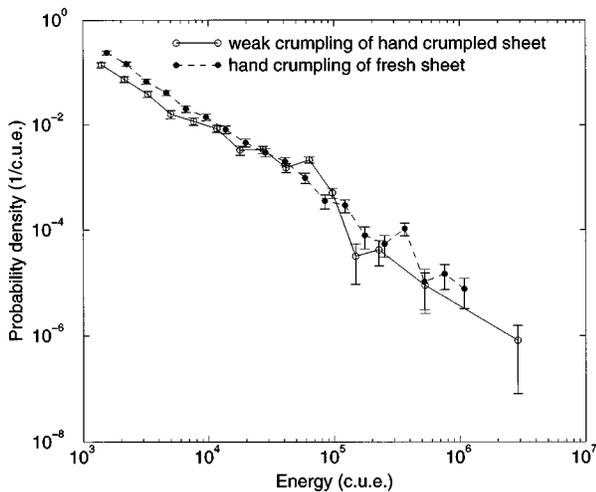


FIG. 9. Weak crumpling of a previously hand-crumpled sheet compared to the sum of three strong crumplings by hand. In both cases we observe that larger events are suppressed more strongly than in cylindrical crumpling.

difference between the buckling metastable and final states is converted into sound, the pulse energies may be reflective of the distribution of the elastic energy stored in and around the facets. If we vary the length scale of an elastic sheet with a constant shape, the energy of bending scales as L^0 and the energy of stretching scales as L^2 where L is the length. If the energy were primarily stored in bending, the energy stored in a facet will have no direct dependence on the area of the facet. However, it has been proposed that when the configuration of an elastic sheet minimizes the sum of bending and stretching energies deformation can isolate itself in temporary ridges (a purely elastic phenomenon distinct from the permanent creases, although it is possible that the extreme deformation at a ridge may produce a crease) with energy scaling as $L^{1/3}$ [23,24]. If the energy emitted during the shift between two stable configurations scaled as weakly as $L^{1/3}$ this could explain our lack of observed finite size dependence; this is plausible if the surface can be understood as an interacting network of ridges as considered in [24]. If it were the case that facet size is not responsible for most of the variation in energy, it is possible that the range of facet geometries, particularly ridge angles, is responsible for the wide range in event energies we observe. It is possible that we observe a small number of very large events only in the earliest stages of crumpling and in the weak crumpling of an initially flat sheet because the existence of an extensive crease network in other situations might limit the range of facet shapes. Whereas a flat or nearly flat sheet forms very sharp cones when stress is applied at the edges (try it), a sheet with a crease network is likely to deform by bending at the creases instead, suppressing facet configurations that may produce high-energy events.

The fact that the oscillation frequency and decay time of ringdowns depends on the type of paper and appears to be the same with both the standard 11-kHz sample rate used for standard recordings and the 44-kHz sample rate used for reference recordings indicates that the ringdowns are a property more of the paper than of the recording system. However, it is interesting that oscillation frequency of pulses does not depend strongly on the pulse energy or the degree of crumpling of the sheet. A possible explanation is that the buckling of a facet concentrates energy into a small area. Such a process would halt at a length scale set by the thickness of the paper, disturbing the surface with a wave number insensitive to facet size and hence little variation in the frequency of oscillation.

Although we see a pulse energy distribution very similar to those seen in avalanche models for which the mechanism of self-organized criticality has been proposed [25], self-organized criticality does not appear to explain our observations. First, our acoustic records and observations indicate that our sound pulses are discrete impulsive events with oscillating exponential decay, not the prolonged noise bursts with a broad power spectrum that are produced by avalanches. Also, since we see no evidence for time evolution in the energy distribution, it does not appear that our power law scaling is a result of evolution to a critical state as is essential to self-organized criticality. It is likely that a new mechanism must exist to explain scaling in our pulse energy distribution.

Many variables were left unexplored in our experiments, and the fact that the ratio of sheet length to thickness is on

the order of 10^3 suggests that finite size effects should be accessible to experiment. Future experiments could probe smaller length scales and gathering better statistics could reveal features of the energy distribution that we were not able to see; mechanization of the crumpling process, made possible by the cylindrical geometry, could be useful towards this end.

In our experiments we have found that the crumpling of paper generates acoustic pulses with a distribution in energy that varies nonexponentially over at least three orders of magnitude and compatible with power law scaling over at least two. We also find that the pulse distribution appears to vary little over time or change in the length scale. Our use of a cylindrical geometry for strong and weak crumpling makes

it possible to crumple paper by a process that is both mathematically and practically well defined, providing a handle for mechanization and theory. However, we do find that cylindrical crumpling may produce a different experimental pulse energy distribution than hand crumpling, perhaps because cylindrical crumpling is fundamentally anisotropic [26].

This project was supported by DOE Grant No. DE-FG02-88-ER45364 and NSF Grant No. DMR-9419506. We thank Wolfgang Sachse for allowing us access to an anechoic chamber, Naresh Kannan for logistic support and many good discussions, as well as helpful discussions with Karin Dahmen, Olga Perković, and Eric Kramer.

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 [26] More information about this research, including audio samples of crumpling paper, can be found on the World Wide Web, URL <http://www.msc.cornell.edu/~houle/crumpling/>