

Generating random walks

(Sethna, "Entropy, Order Parameters, and Complexity", ex. 2.5)

Write a routine `RandomWalk[N,d]` to generate an N-step random walk in d dimensions, with each step uniformly distributed in $(-1/2,1/2)$ in each dimension.

For $d>1$, generate the steps first as an $N \times d$ array, and then do a cumulative sum with `Accumulate`.

Define $d=1$ separately, just generating N random numbers and then accumulating. (*Mathematica* treats an $N \times 1$ array as a list of lists, each with one element.)

```
RandomWalk[N_, 1] := (steps = ...; Accumulate[steps])  
RandomWalk[N_, d_] := (steps = ...; ...)
```

ListPlot some one dimensional random walks versus step number, for $N=10000$ steps. ListPlot some two-dimensional random walks with $N=10000$ steps.

Set `AspectRatio`→`Automatic` to make the x and y scales the same.

```
ListPlot[Table[RandomWalk[...], {i, 1, 3}], Joined → True]
```

```
ListPlot[Table[...], {i, 1, 3}], Joined → True, AspectRatio → Automatic]
```

Now write a routine `Endpoints[W, N, d]` that just returns the endpoints of W random walks of N steps each in d dimensions. (No need to use `Accumulate`; just use `Total`. If you generate a 3D array of size (N, W, d) , `Total` will sum over the N steps of each walk. Again, define $d=1$ separately.

```
Endpoints[W_, N_, 1] := Total[...]  
Endpoints[W_, N_, d_] := Total[...]
```

ListPlot the endpoints of 10000 random walks of length 10, together with the endpoints of 10000 random walks of length 1, appropriately setting the `AspectRatio` (and, of course, without `Joined`→`True`). Discuss.

```
ListPlot[{Endpoints[...], Endpoints[...]}, AspectRatio → Automatic]
```

Find σ for an N-step random walk with uniform step sizes in $(-1/2,1/2)$. Compare the normalized histogram of 10000 endpoints with a normalized Gaussian of width σ , for $N=1, 2, \text{ and } 5$.

```
hist =  
  Histogram[{Endpoints[...], Endpoints[...], ...}, {0.1}, "ProbabilityDensity"]  
Gauss[ $\sigma$ _] := (1 / Sqrt[...]) Exp[...]  
 $\sigma$ [N_] := Sqrt[...]  
Show[hist, Plot[Gauss[ $\sigma$ [1]], {x, -3  $\sigma$ [1], 3  $\sigma$ [1]}], Plot[...], ...]
```