8:30 Opening Remarks: Peter Lepage

Chair: Bruce Patton
8:40 Allan Griffin, University of Toronto
Many body physics in the 1960s: A Golden Age
9:05 Anthony Leggett, University of Illinois
Quantum dissipation in the 80s and today
9:35 Jason Ho, The Ohio State University
New challenges and future opportunities in cold atoms

10:05 Break

Chair: Alexei Maradudin
10:30 David Mermin, Cornell University
Things I wish they had told me about Shor’s factoring algorithm
11:00 Eric Siggia, Rockefeller University and Cornell University
Unconventional physics problems from biology
11:30 Alexis Baratoff, University of Basel
Reduction of friction in atomic-scale contacts — A novel model

12:00 Lunch

Chair: Joseph Serene
1:30 Gerd Schön, University of Karlsruhe
Solid-state qubits and decoherence
2:00 Mohit Randeria, The Ohio State University
High T, superconductivity: Where are we after 20 years?
2:30 Sudip Chakravarty, University of California, Los Angeles
Entanglement

3:00 Break

Chair: Aashish Clerk
3:30 Jan Von Delft, University of Munich
Mesoscopic to universal crossover of transmission phase of multi-level quantum dots
4:00 Bertrand Halperin, Harvard University
Number parity in superconductors and the f=5/2 fractional quantized Hall state
4:35 Walter Kohn, University of California, Santa Barbara
The Power of the Sun (a movie about 1 hour long)

6:30 Dinner
Fifty year ago, in 1957, two special things happened:
1. Vinay started his Ph. D. research.
2. The BCS long paper was published.

The anniversary of the famous BCS paper reminded me that in many ways, 1957 was an absolutely pivotal year in the history of how we think about many body systems in condensed matter physics. This will be the subject of my talk today on this special occasion. Vinay said that he wanted us to talk about current research, not the old days. Since I took mandatory retirement in 2004, I feel that I am now his senior and can go against the wishes of my supervisor.
In a nutshell, I will argue that in the years 1957-1960, the way we think about interacting many body systems underwent a revolution. In theoretical physics, it compares with the development of quantum mechanics in the golden period of 1925-1927.

The magic year was precisely 50 years ago, in 1957. A almost endless number of papers were published in a single year that, looking back, lead to a paradigm shift. Many classic problems that had been nagging theorists since the 1930s were completely solved. Perhaps more important, we learned new words and a new language to describe interacting systems.

We developed a new set of criteria that defined what we meant when we said we understood a phase of condensed matter.
Nagging problems in solid state physics:

1. **Theory of metals.**
   A good description was based on a Fermi gas of independent electrons moving in the periodic lattice of ions. This was the famous **Sommerfeld-Bloch picture developed around 1930.** Why could one forget about the strong Coulomb interactions between electrons? More precisely, why could we neglect the so-called **correlation energy?**

2. **Theory of superconductivity.**
   There were a lot of useful **phenomenological theories**, such as the **Ginzburg-Landau (GL) theory** (1950) and the **London equations** (1935). But there was no underlying **microscopic basis** for these theories, or any idea of the cause of this strange **phase transition.**
What did theorists do in the period 1950-1956?

They knew that one had to include interactions somehow. The main approach was to introduce some correlations into the many particle wave function for the ground state of the system, and treat it variationally.

This precluded any ability to deal with a phase transition, since the concept of an order parameter was still not really understood. Moreover it was difficult to build an order parameter into a quantum mechanical wavefunction!

The greatest success of this approach was with interacting Bosons. Feynmann (1954) introduced a variational groundstate wavefunction for a Bose liquid. In 1956, Oliver Penrose and Onsager used this kind of wavefunction to define what a Bose condensed phase was and estimated the condensate fraction in superfluid He⁴ to be 8%.
David Bohm and his graduate student David Pines tried to develop a theory of the collective modes in an interacting electron gas. Their approach was to introduce the coordinates which would describe the these collective degrees of freedom into the quantum mechanical equations of motion describing density and velocity variables.

In a series of seminal papers (1951-1953), Bohm and Pines developed what we now call the RPA theory of plasmon oscillations in metals, separate from the single particle excitations which now interacted by a weak screened Coulomb interaction. However, the Bohm-Pines theory was controversial, because it was felt that there was an overcounting problem with the degrees of freedom. The challenge was to justify their theory.
The dense electron gas problem solved in 1957!

The Bohm-Pines RPA theory became a challenge to theorists. The problem was solved by the work of many people, but especially in a paper by Gell-Mann and Brueckner published the magic year of 1957. They summed up the leading order Feynmann diagrams in a dense electron gas and evaluated the correlation energy in the ground state, getting agreement with the Bohm-Pines results.

Within months, the explicit role of the plasmons vs particle-hole excitations was elucidated by GB working with Robert Brout and Sawada.

This success made diagrammatic methods and Green’s functions very popular, as did a flood of papers in 1957 using and developing these methods.
Papers on interacting electron systems in 1957

Landau (1956 & 1957)
Beliaev
Galitskii
Migdal
Hubbard
Nozieres
Gell-Mann & Brueckner
Anderson
Brout
Bogoliubov

Goldstone
Thouless
Hugenholtz

...
A new language developed after 1957

The structure of correlation functions in space and time
Single particle Green’s functions
Linear response functions and Kubo formulas
Order parameters
Broken symmetry and Goldstone modes
Renormalized interactions and vertex functions
Propagator renormalization
Collective modes vs single particle excitations
Collisionless vs hydrodynamic regions

The age of theoretical summer schools

This new gospel was transmitted at many summer schools on the many body problem in the following years, starting with Les Houches in 1958 (Nozieres). In the 1960s, Vinay was a frequent lecturer on Fermi liquid and BCS theory.
In 1957, Landau formulated a theory of an interacting Fermion gas, based on the assumption that a **sharp Fermi surface** well known in non-interacting Fermi gases would **persist** even in the presence of strong interactions. Finally we had a **definition** of what a **metal** was! Now called a **normal Fermi liquid**.

However, it was a phenomenological theory. It stimulated a whole series of papers in the next few years with the aim to give a precise **microscopic basis of Landau theory** using Green’s functions and many body theory. Key papers were by **Kohn**, **Luttinger** and **Ward**, as well as by **Nozieres**.
Bardeen, Cooper and Schrieffer finally solved the mystery of superconductivity based on the idea of Cooper pairs. Their theory was magnificent physics but “old fashioned” in the sense that it was based on an explicit many particle ground state wavefunction and excited states.

Within a few months, Gorkov showed that the BCS theory corresponded to a simple mean field theory but besides the usual Hartree-Fock terms, one had to also include a off-diagonal mean field describing the effect of “a sort of Bose condensate of Cooper pairs” (to quote Gorkov)

It is curious that it would only be in 1985 that Leggett re-emphasized that BCS should be thought in terms of such a Bose condensate of bound pairs
The key new concept introduced by Gorkov introduced was an anomalous off-diagonal single particle propagator related to bound pair formation.

\[ G_{12} \propto \langle \psi(r,t)\psi(r',t') \rangle \]

in addition to normal diagonal propagator

\[ G_{11} \propto \langle \psi^+(r,t)\psi(r',t') \rangle \]

This allowed Gorkov to re-derive the whole complex BCS theory in a 3 page paper. The condensate mean field is

\[ \Delta \propto -U \langle \psi \psi \rangle \]

and gives rise to the energy gap in the BCS quasiparticle spectrum.
Landau’s group in Moscow, 1956.

A. Abrikosov, I. Khalatnikov, L. Landau, E. Lifshitz.
L. Pitaevskii, I. Dzyaloshinskii.

The famous AGD book on was being hatched by these people!
Another school similar to the Landau powerhouse grew out of the famous paper by Martin and Schwinger (1959).

A whole group of graduate students came out of Harvard in the early 1960s using the mantra of thermal Green’s at imaginary frequencies. This work was captured best by the classic book by Kadanoff and Baym (1962) on Green’s function techniques for non-equilibrium problems.

Vinay received his Ph.D. in 1960 and joined Cornell in 1962. He mastered the Martin-Schwinger techniques and used them on a variety of novel problems in the 1960s in the area of superconductivity.
Moscow 1965. USSR-USA Seminar.
The Gorkov formulation of BCS led to a whole bunch of extensions to new kinds of superconductors. The subject was pounced on by the Landau school!

- **Strong - coupling theory** - Eliashberg, 1960
- **Gapless superconductivity** - Abrikosov and Gorkov, 1960
- **Derivation of the GL theory of Type-II superconductors** - Gorkov, 1959
- **Josephson effect** - Josephson, Ambegaokar and Baratoff
- **Collective oscillations of the Cooper pair condensate**
  - Anderson (1958), Bogoliubov, Ambegaokar and Kadanoff
- **p-wave and d-wave pairing** - found many years later in superfluid He\(^3\) and in high T\(_c\) superconductors.
The idea was to split the quantum field operator into two parts

$$\psi = \langle \psi \rangle_{BS} + \tilde{\psi}$$

With the broken symmetry order parameter separated out, the **noncondensate part** could be treated by regular many Body diagrammatic techniques.

Beliaev’s work was done **down the hall** from Gorkov, who was introducing the analogous formalism for superconductors.

Beliaev’s approach had great success in formulating the dynamical role of a Bose condensate in a liquid, even if calculations could not be made. The **conceptual basis** was settled in the period 1957-1964, with key papers by Gavoret and Nozieres (1964) and Hohenberg and Martin (1965).
Work on the many body theory of interacting Fermi and Bose quantum gases started in 1957 is now having a revival because of experiments in ultracold atomic gases.

Many of these original calculations could never be tested until ultracold gases could be produced in the lab, starting in 1995.

The whole field of many body physics is being pushed along with these new studies. They are the dream system of many body theorists, since the interaction strength and density can be adjusted at will by turning knobs!
The pioneering work of BCS- Gorkov on superconductivity and Beliaev on BEC has been beautifully joined in the last few years in **two component atomic Fermi gases**. As first studied in detail by **Leggett** in 1980, by increasing the attractive interaction between Fermions, one has a **smooth crossover** from a standard weak coupling superconducting BCS phase to an interacting gas of Bosonic molecules.

This **BCS-BEC crossover** is a hot topic in current research in ultracold atomic Fermi gases, verifying work at **finite** temperatures done many years ago by **Randeria** (1993, 1997) and **Nozieres** (1985).
The work that solved the problem of normal metals and superconductors was a magnificent achievement. It was part of the revolution in our understanding of many body problems starting in 1957. They should be celebrated as a high point in the history of theoretical physics. Vinay and many of us had the privilege of building on this base.

In recent years, one has a tendency of referring to normal metals and the BCS superconductors as boring, as we grapple with highly-correlated metals and high temperature superconductors. Perhaps it is a sort of jealousy, as we try to find an equivalent set of powerful ideas that first emerged in 1957. We should also remember that these theories gave us the language that we use now to discuss all many body systems, even these unsolved exotic phases!
Some "Hybrid" Optical-Solid State Qubits

RF SQUID
(Delft, SUNY, Saclay...)

Cavity QED
(ENS)

Superconducting qubit coupled to QED cavity
(Yale)

Surface plasmon
(Geneva)
\[ \Psi \sim |1\rangle |\Phi_1\rangle + |2\rangle |\Phi_2\rangle \]

\[ |1\rangle \sim (\exp \ i \alpha \theta)^{n/2} \]

\[ |2\rangle \sim (\exp \ -i \alpha \theta)^{n/2} \]

(no of Cooper pairs \( \approx 10^6 - 10^9 \))

\( \langle 1|2 \rangle \ll 1 \Rightarrow 55 \text{ DOF's strongly involved} \)

\( \Psi \sim a \text{vac} + b l_\gamma \)

but:

"Photon only confined to cavity beam, strongly interacting with electrons in wells"

\( \Rightarrow \Psi \sim a \text{vac} |1_e\rangle + b l_\gamma |2_e\rangle \)

\( \langle 1_e|2_e \rangle \ll 1 \?? \)
What is a surface plasmon?

Total no. of electrons displaced into surface zone:
\[ n \approx (\omega/\omega_p) \cdot \left(2\pi hL/\lambda\right) \approx 300 \] for Fasel et al. expr.

- Bulk plasma freq.
- Lateral origin of S.P. wave fronts
- Free-space wavelength of photon

(also: EPR-Bell expr. with converters for each photon)

requires
\[ \varepsilon_1 + \varepsilon_2 < 0 \]

i.e. metal-dielectric interface
How to define "degree of involvement"?

In absence of S.P.: \( \Psi = \Psi_0(1,2) \)

in presence of S.P.
\( \Psi = \Psi_\gamma = \Psi_\gamma(1,2) \)

how to compare \( \hat{\Psi}_\gamma^{(2)} \) and \( \hat{\Psi}_0^{(2)} \)?

\[ J = 1 - F = 1 - \left\{ \text{Tr} \left( \sqrt{\hat{\Psi}_\gamma} \hat{\Psi}_0 \sqrt{\hat{\Psi}_\gamma} \right)^{1/2} \right\} \]

Problem: if
\[ \hat{\Psi}_\gamma = (1-\varepsilon) \hat{\Psi}_0 + \varepsilon \hat{\Delta} \]

then \( J \sim \varepsilon^2 \approx \varepsilon^2 \).

So: use simply

\[ J = \varepsilon \]

In special case
\[ |\gamma\rangle = (\alpha \hat{\Omega}_1^\dagger \times \hat{\Omega}_2^\dagger + \beta \hat{\Omega}_1 \times \hat{\Omega}_2^\dagger) |0\rangle \]

\[ \langle 0| \hat{\Omega}_3 |0\rangle = 0, \quad \langle 0| \hat{\Omega}_3^\dagger \times \hat{\Omega}_3 |0\rangle = \delta_{3\gamma} \]

df. simiplicity

\[ J = |\beta|^2 \]

The 4K question: How "involved" on the metal?

i.e. \( J \) is "more" of exact? attributable to metal.

metallic electrons?
If excitation of metal can be expressed in terms of normal modes \((k,n)\):

\[ J = \frac{1}{2} \sum \left\{ |X_{kn}|^2 / \langle |X_{kn}|^2 \rangle_0 \right\} \]

\(X_{kn}\) = amplitude of excit. of mode \(k,n\) in S.P.,

\(\langle |X_{kn}|^2 \rangle_0\) = mean-square zero-point amplitude

**A SIMPLE MODEL: "LONDON" DYNAMICS**

\[ \text{df. of model: } j(r,t) = -\varepsilon \omega_p A(r,t) \]

\(e\) = electron

\(\omega_p\) = bulk plasma freq.

\(\varepsilon (m e^2/m_0)^{1/2}\)

In this model there is only one longitudinal \((L)\) and one transverse \((T)\) mode for each \(k\), with freq.

\[ \omega_L = \omega_p \]

\[ \omega_T = (\omega_p^2 + c^2k_0^2)^{1/2} \]

Express contrib. to \(J\) in terms of values of \(A(r,t)\)

junk inside metal in S.P., \(A_0\):

**L case:**

\[ J \sim \frac{\varepsilon_0 \omega_p}{\hbar_k} \sum_k |A_k|^2 \quad A_k \sim A_0 k_0^{-1} \quad \omega_k \sim k_0 c \]

so,

\[ J_0 \sim \frac{\varepsilon_0}{2\pi\hbar_k} \left( \frac{\omega_p}{k_0} \right) |A_0|^2 \]
Transverse contribution:

\[ A_L \propto A_0 \left( k(w) + ik \right)^{-1} \]

\[ \Xi = (\omega_p^2 - \omega^2)^{1/2} / \omega \]

so:

\[ J_T = \frac{\epsilon_0 e^2}{2 \pi k} |A_0|^2 \int_0^{\infty} \frac{dk}{k^2 (k^2 - \omega^2)} (\omega < \omega_p) \]

Avoid log's divergence (a) by cutoff at \( k = k_c \), or
(b) by noting that most of "involvement" of metal for \( \omega \approx \omega_p \) is simply that of EM DOF: The contribution of electronic DOF's is only a factor \( \propto \omega_p^2 / \omega_c^2 \). So, multiplying integrand by this factor,

\[ \tilde{J}_T \propto \frac{\epsilon_0 e^2}{2 \pi k} |A_0|^2 F(\omega / \omega_p) \]

\[ \sim 1 \text{ unless } \omega / \omega_p \]

Since generally \( e k_c > \omega_p \), \( J_c \ll J_T \) and

\[ J \leq \left( \epsilon_0 e^2 / 2 \pi k \right) |A_0|^2 \]

(recall: \( A_0 \) is EM VP of SP "just inside" metal, i.e. just beyond surface layer

\[ \cdots \cdots \cdot \]

\[ \lambda_c \]

[Will verify below that this is correct 0. of m. also for more realistic models]
General Expression For \( J \):

\[
J_L = \frac{\varepsilon_0}{2\pi \hbar c} (\omega, l) |A_0|^2 \Phi(\omega)
\]

\[
J_T = \frac{\varepsilon_0 c}{4\pi} \Phi(\omega)
\]

where dimension factors \( \Phi(\omega), \Phi(\omega) \) are given by:

\[
\Phi(\omega) \equiv \omega^{-1} \omega^+ |\epsilon_0(\omega)|^2 \cdot 2 \int_0^\infty \frac{d\omega'}{\omega'^2 - \omega^2} \text{Im} \left( -\frac{1}{\epsilon_0(\omega')^2} \right)
\]

\[
\Phi(\omega') \equiv \Phi(\omega, l) \equiv 2 \epsilon_0 \int_0^\infty \frac{d\omega'}{\omega'^2 - \omega^2} \text{Im} \left\{ \frac{|\kappa(\omega', l)|^2 - \kappa^2(\omega', l)^2}{\kappa(\omega', l)} \right\}
\]

\[
\kappa(\omega, l) \equiv \frac{c}{\epsilon_0} (-\epsilon_0(\omega) \omega^2 + c^2 k_{\parallel}^2)^{1/2}
\]

*Complex known propagation depth of any \omega.*

Note: both \( \Phi(\omega) \) and \( \Phi(\omega') \) are of the form

\[
\text{Conc.} \int_0^\infty \frac{F(\omega')}{\omega'^2} d\omega'
\]

How to handle divergence from \( \omega' = \omega \)? Replace monochromatic wave by wave packet. E.T.

\[
A_0 \equiv A_0(t) = \mathcal{A}_0 f(t).
\]

\[
\int_0^\infty |f(t)|^2 dt = 1.
\]
For a wave packet, $A_0(t) \sim f(t)$,

$J \equiv J(t) = J_{mv}(t) + J_{imv}(t)$

$J_{mv}(t) = J_{mv} \frac{|f(t)|^2}{t}$

cont'd of non-singular term of $C^1$-current

$J_{imv}(t) = \frac{c \xi_o |A_o|^2}{t} (-\ln k(\omega_o)) Q(t)$

$Q(t) \equiv \int_{-\infty}^{t} |f(t)|^2 dt$

$\Rightarrow J_{imv}(t)$ is simply prob. that excit. has been absorbed up to time $t$.

What about the "reversible" term? For long time one must evaluate

$\int_{0}^{t} \frac{dw'}{\omega'^2 - \omega^2} \ln\left(\frac{1}{\xi(\omega')}\right) = \text{sing. cont.}$

lots of sum rules, e.g.

$\frac{2}{\pi} \int \omega' d\omega' \ln\left(-1/\xi(\omega')\right) = \omega^2$

$\frac{2}{\pi} \int \frac{d\omega'}{\omega'} \ln\left(-1/\xi(\omega')\right) = 1$
so, generically, expect $\int w \omega^{-3}$ and hence $\Phi(\omega) \approx 1$.

However, a surprise:

for simple Drake model ($\sigma(\omega) = \epsilon_0 \omega^2/(1-i\omega c)$) we can evaluate $\Phi(\omega)$ for arbitrary $\omega$. Then if we take the limits

$$2^{-1} \ll \omega \ll c,$$

(but no restriction on ratio of small quantities $\omega/\epsilon_0$, $1/\omega c$) we find

$$\Phi(\omega) = 1 - \frac{\omega_p}{2\pi \omega^2 c} + o(1/\omega c)$$

For currently realistic exoplanet parameters, $\omega_p/2\pi \omega^2 c \ll 1$. But much not is so $\Rightarrow J_{nw}(k)$ can be $-ve$!

**OK, but only physically meaningful quantity is**

$$J(\omega) \equiv J_{nw}(k) + J_{nsw}(k),$$

and this is always $+ve$.

In any case, we expect

$$\Phi(\omega) \leq 1,$$

$$\Phi(\omega) \leq 1$$

and so, just as in simple “london” model,

$$J \leq (\epsilon_0 \omega^2/\pi c) |A|^2$$

Now the crunch: determine $A_0$ from the matching conditions at 1-2 interface and the consideration that total (external) field en. $\approx$ thr.
The net result: for a S.P.,
\[ |A_0|^2 \approx \frac{k e_1}{\varepsilon_0 c \varepsilon_2^e} \left(1 + \varepsilon_2^e\right)^{1/2} \]
so
\[ J \approx \frac{1}{\pi} \frac{e_1}{\varepsilon_2^e} \frac{1}{\left(1 + \varepsilon_2^e\right)^{1/2}} \]

But for \( \omega < \omega_p \) (empirical, must check!)
\[ e_1 \sim 1, \quad e_2^e \sim -\frac{\omega_p^2}{\omega^2} \]
so
\[ J \approx (\omega/\omega_p)^3 \]

In expt. of Farhi et al., metal is Au (\( \omega_p \approx 8 \text{ eV} \))
and \( \omega \) is in Telemann band (1500 nm \( \approx 0.75 \text{ eV} \)), so
\[ J \approx 10^{-3}. \]

How can this be? \[ \begin{array}{ccc}
+ & + & + \\
\hline
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } \\
\text{ } & \text{ } & \text{ } \\
\end{array} \]
\[ \sim 300 \text{ e}^{-}\text{'s in total!} \]

Regard surface layer as II-plate capacitor, calculate zero-point fluctuation in absence of S.P. :
\[ Q^2/2e = \frac{1}{4} e_k \omega_p \Rightarrow N \sim \left(\frac{e_0 k e^2 L^2}{\varepsilon_2^e} \varepsilon_0 \varepsilon_2^e\right)^{1/2} \sim L/\alpha \]
\[ \Rightarrow \]
\[ N_{z1} \approx 10^7 ! \]
Things I wish they had told me about Shor’s factoring algorithm

Vinay Ambegaokar’s Retirement Symposium
Cornell, June 16, 2007
Question:
What has quantum mechanics to do with factoring?

Answer:
Nothing!

But quantum mechanics is good at diagnosing periodicity, which (for purely arithmetic reasons) helps in factoring.
Factoring $N = pq$, where $p$ and $q$ are huge (e.g. 300 digit) primes, follows from ability to find smallest $r$ with $a^r \equiv 1 \pmod{N}$ for integers $a$ sharing no factors with $N$.

$a^x \pmod{N}$ is periodic with period $r$.

Pick random $a$. Use quantum computer to find $r$.

Pray for two pieces of good luck.
Quantum computer gives least $r$ with $a^r - 1$ divisible by $N = pq$

**First piece of luck:** $r$ even.
Then $(a^{r/2} - 1)(a^{r/2} + 1)$ is divisible by $N$, but $a^{r/2} - 1$ is not,

**Second piece of luck:** $a^{r/2} + 1$ is also not divisible by $N$.
Then product of $a^{r/2} - 1$ and $a^{r/2} + 1$ is divisible by both $p$ and $q$ although neither factor is divisible by both.

Since $p, q$ primes, one factor divisible by $p$ and other divisible by $q$.
So $p$ is greatest common divisor of $N$ and $a^{r/2} - 1$
and $q$ is greatest common divisor of $N$ and $a^{r/2} + 1$

*FINISHED!*
Finished, because:


2. If \(a\) is picked at random, an hour’s argument shows that the probability is at least 50\% that both pieces of luck will hold.
Amazing (but wrong):

[After the computation] the solutions — the factors of the number being analyzed — will all be in superposition.


[The computer will] try out all the possible factors simultaneously, in superposition, then collapse to reveal the answer.

— Ibid.

Unexciting but correct:

A quantum computer is efficient at factoring because it is efficient at period-finding.
Next question: What’s so hard about period finding?

Given graph of $\sin(kx)$ it’s easy to find the period $2\pi/k$. Since no value repeats inside a period, $a^x \pmod{N}$ is even simpler.

What makes it hard:

Within a period, unlike the smooth, continuous $\sin(kx)$, the function $a^x \pmod{N}$ looks like random noise.

Nothing in a list of $r$ consecutive values gives a hint that the next one will be the same as the first.
PERIOD FINDING WITH A QUANTUM COMPUTER

Represent $n$ bit number

$$x = x_0 + 2x_1 + 4x_2 + \cdots + 2^{n-1}x_{n-1} \quad (\text{each } x_j \ 0 \text{ or } 1)$$

by product of states $|0\rangle$ and $|1\rangle$ of $n$ 2-state systems (Qbits):

$$|x\rangle = |x_{n-1}\rangle \cdots |x_1\rangle |x_0\rangle$$

Classical or Computational basis.
Qbits, not qubits because:

1. Classical two state systems are Cbits (not clbits)
2. Ear cleaners are Qtips (not Qutips)
3. Dirac wrote about q-numbers (not qunumbers)

(q-bit awkward: 2-Qbit gate OK; 2-q-bit gate unreadable.)
Represent function $f$ taking $n$-bit to $m$-bit integers by a linear, norm-preserving (unitary) transformation $U_f$ acting on $n$-Qbit input register and $m$-Qbit output register:

$$U_f |x\rangle|0\rangle = |x\rangle|f(x)\rangle.$$
QUANTUM PARALLELISM

\[ U_f |x\rangle |0\rangle = |x\rangle |f(x)\rangle \]

Put input register into superposition of all possible inputs:

\[ |\phi\rangle = \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq x < 2^n} |x\rangle \]

\[ = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \cdots \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle). \]

Applying linear \( U_f \) gives

\[ U_f (|\phi\rangle |0\rangle) = \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq x < 2^n} |x\rangle |f(x)\rangle. \]
\[ U_f(|\phi\rangle|0\rangle) = \left(\frac{1}{\sqrt{2}}\right)^n \sum_{0 \leq x < 2^n} |x\rangle|f(x)\rangle. \]

Special form when \( f(x) = a^x \pmod{N} \):

\[ \sum_{0 \leq x < 2^n} |x\rangle|a^x\rangle = \sum_{0 \leq x < r} \left( |x\rangle + |x + r\rangle + |x + 2r\rangle + \cdots \right)|a^x\rangle \]

Measuring output register leaves input register in state

\[ |x\rangle + |x + r\rangle + |x + 2r\rangle + \cdots \]

for random \( x < r \).
THE QUANTUM FOURIER TRANSFORM (QFT)

\[ V_{FT} |x\rangle = \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq y < 2^n} e^{2\pi i y / 2^n} |y\rangle \]

Acting on superpositions, \( V_{FT} \) Fourier-transforms amplitudes:

\[ V_{FT} \sum \alpha(x) |x\rangle = \sum \beta(x) |x\rangle \]

\[ \beta(x) = \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq z < 2^n} e^{2\pi i x z / 2^n} \alpha(z) \]

If \( \alpha \) has period \( r \), as in \( |x\rangle + |x + r\rangle + |x + 2r\rangle + \cdots \), then \( \beta \) is sharply peaked at integral multiples of \( 2^n / r \).
**HO-HUM!**

$V_{FT}$ is *boring*:

1. Just familiar transformation from position to momentum representation.

2. Everybody knows Fourier transform sharply peaked at multiples of inverse period.
But $V_{FT}$ is not ho-humish because:

1. $x$ has nothing to do with position, real or conceptual. 
   $x$ is arithmetically useful but physically meaningless:
   
   $$x = x_0 + 2x_1 + 4x_2 + 8x_3 + \cdots,$$
   
   where $|x_j\rangle = |0\rangle$ or $|1\rangle$ is state of $j$-th 2-state system.

2. *Sharp* means sharp compared with resolution of apparatus. 
   The period $r$ is hundreds of digits long. 
   Need to know $r$ exactly — every single digit. 
   Error in $r$ of 1 in $10^{10}$ messes up almost every digit.
\[
\mathbf{V}_{FT}(\lvert x \rangle + \lvert x + r \rangle + \lvert x + 2r \rangle + \cdots) = \\
= \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq y < 2^n} \left( 1 + \alpha + \alpha^2 + \alpha^3 + \cdots \right) e^{2\pi i y/2^n} \lvert y \rangle,
\]

\[\alpha = \exp\left(2\pi i y/(2^n/r)\right).\]

Sum of phases \(\alpha\) sharply peaked at values of \(y\) as close as possible to (i.e. within \(\frac{1}{2}\) of) integral multiples of \(2^n/r\).

**Question:** How sharply peaked?

**Answer:** Probability that measurement of input register gives such a value of \(y\) exceeds 40%.
Good (> 40%) chance of getting integer $y$ within $\frac{1}{2}$ of $j(2^n/r)$ for some (more or less) random integer $j$.

Then $y/2^n$ is within $1/2^{n+1}$ of $j/r$.

**Question:** Does this pin down unique rational number $j/r$?

**Answer:** Yes, if $2^n > r^2$.

*Since $r < N$, input register must be large enough to represent $N^2$. Must contain at least $N$ periods.*

Then have 40% chance of learning a *divisor* $r_0$ of $r$.

($r_0$ is $r$ divided by factors it shares with (random) $j$)
Comment:

Should the period \( r \) be \( 2^m \), then \( 2^n/r \) is itself an integer, and probability of \( y \) being multiple of that integer is easily shown to be 1, even if input register contains just a single period.

A pathologically easy case.

Question: When must all periods \( r \) be powers of 2?
Answer: When \( p \) and \( q \) are both of form \( 2^j + 1 \).

(Periods are divisors of \( (p - 1)(q - 1) \).)

Therefore factoring \( 15 = (2 + 1) \times (4 + 1) \) — i.e. finding periods modulo 15 — is not a serious demonstration of Shor’s algorithm.
Some neat things about the quantum Fourier transform

\[ \mathbf{V}_{FT}|x\rangle = \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq y < 2^n} e^{2\pi i xy/2^n} |y\rangle \]

1. Constructed entirely out of 1-Qbit and 2-Qbit gates.
2. Number of gates and therefore time grows only as \( n^2 \).
3. With just one application,

\[ \sum \alpha(x)|x\rangle \longrightarrow \sum \beta(x)|x\rangle , \]

\[ \beta(x) = \left( \frac{1}{\sqrt{2}} \right)^n \sum_{0 \leq z < 2^n} e^{2\pi i x z/2^n} \alpha(z) \]

In classical “Fast Fourier Transform” time grows as \( n2^n \).

But classical FFT gives all the \( \beta(x) \), while QFT only gives \( \sum \beta(x)|x\rangle \).
\[ |x\rangle \]

\[ |x_0\rangle \]
\[ |x_1\rangle \]
\[ |x_2\rangle \]
\[ |x_3\rangle \]
\[ |x_4\rangle \]
\[ |x_5\rangle \]

\[ \mathcal{V}_{FT}|x\rangle \]

\[
\begin{align*}
|0\rangle & \quad \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \\
|1\rangle & \quad \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)
\end{align*}
\]

\[ e^{\pi i n n' / 2} \quad e^{\pi i n n' / 4} \quad e^{\pi i n n' / 8} \quad e^{\pi i n n' / 16} \quad e^{\pi i n n' / 32} \]

\[ |0\rangle |0\rangle, \quad |0\rangle |1\rangle, \quad |1\rangle |0\rangle \text{ invariant}; \quad |1\rangle |1\rangle \rightarrow e^{\pi i / 2^j} |1\rangle |1\rangle \]
A PROBLEM?

Number $n$ of Qbits: $2^n > N^2$, $N$ hundreds of digits.

Phase gates $e^{\pi i \text{nn}'/2^m}$ impossible to make for most $m$, since can’t control strength or time of interactions to better than parts in $10^{10} = 2^{30}$.

But need to learn period $r$ to parts in $10^{300}$ or more!
Question:
    So is it all based on a silly mistake?
Answer:
    No, all is well.
Question:
    How can that be?
Answer:
    Because of the quantum-computational interplay between analog and digital.
Quantum Computation is Digital

Information is acquired \textit{only} by measuring Qbits. The reading of each 1-Qbit measurement gate is only 0 or 1.

The $10^3$ bits of the output $y$ of Shor’s algorithm are given by the readings (0 or 1) of $10^3$ 1-Qbit measurement gates.

There is no imprecision in those $10^3$ readings. The output is a definite 300-digit number.

\textit{But is it the number you wanted to learn?}
Quantum Computation is Analog

Before a measurement the Qbits
are acted on by unitary gates with
continuously variable parameters.

These variations affect the amplitudes
of the states prior to measurement
and therefore they affect the *probabilities*
of the readings of the measurement gates.
So all is well

“Huge” errors (parts in $10^4$) in the phase gates may result in comparable errors in the probability that the 300 digit number given precisely by the measurement gates is the right 300 digit number.

So the probability of getting a useful number may not be 40% but only 39.99%.

Since “40%” is actually “about 40%” this makes no difference.
In fact this makes things even better

Since only top 20 layers of phase gates matter, when \( N > 2^{20} = 10^6 \), running time scales not quadratically but linearly in number of Qbits.
Another Important Simplification

1-Qbit measurements

\[ |x\rangle + |x + r\rangle + |x + 2r\rangle + |x + 3r\rangle + \cdots \]

\[ e^{\pi i n n'/2} + e^{\pi i n n'/4} + e^{\pi i n n'/8} + e^{\pi i n n'/16} + e^{\pi i n n'/32} \]
Another Important Simplification

1-Qbit measurements

\[ |x\rangle + |x + r\rangle + |x + 2r\rangle + |x + 3r\rangle + \cdots \]

\[ e^{\pi iy_0 n/2} e^{\pi iy_0 n/4} e^{\pi iy_0 n/8} e^{\pi iy_0 n/16} e^{\pi iy_0 n/32} \]

You don’t need anything but 1-Qbit gates!
References:

Cornell lecture notes
[Google: mermin CS483]

Quantum Computer Science
N. David Mermin
Cambridge University Press
August 2007 (I hope)
Quantum Versus Classical Programming Styles

Question: How do you calculate $a^x$ when $x$ is a 300 digit number?
Answer: Not by multiplying $a$ by itself $10^{300}$ times!

How else, then?

Write $x$ as a binary number: $x = x_{999}x_{998} \cdots x_2x_1x_0$.

Next square $a$, square the result, square that result . . . ,
getting the 1,000 numbers $a^{2^j}$.

Finally, multiply together all the $a^{2^j}$ for which $x_j = 1$.

$$\prod_{j=0}^{999} (a^{2^j})^{x_j} = a^{\sum_j x_j2^j} = a^x$$
Classical: Cbits Cheap; Time Precious

\[ a^x = \prod_{j=0}^{999} \left( a^{2^j} \right)^{x_j} \]

Once and for all, make and store a look-up table:

\[ a, a^2, a^4, a^8, \ldots, a^{2^{999}} \]

A thousand entries, each of a thousand bits.

For each \( x \) multiply together all the \( a^{2^j} \) in the table for which \( x_j = 1 \).
Quantum: Time Cheap; Qbits Precious

Circuit that executes

\[ a^x = \prod_{j=0}^{999} \left(a^{2^j}\right)^{x_j} \]

is not applied \(2^n\) times to input register for each \(|x\rangle\).
It is applied \textit{just once} to input register in the state

\[ |\phi\rangle = \left(\frac{1}{\sqrt{2}}\right)^n \sum_{0 \leq x < 2^n} |x\rangle. \]

So after each conditional (on \(x_j = 1\)) multiplication by \(a^{2^j}\) can store \(a^{2^j} = a^{2^{j+1}}\) using same 1000 Qbits that formerly held \(a^j\).
Evolution of antibiotic resistance by whole genome resequencing
Staphylococcus aureus and British elections I

FURIOUS relatives of a pensioner who had to have a leg amputated below the knee after contracting MRSA are taking legal action. John Drever, 76, survived a quadruple heart bypass operation only to be struck down by the virulent superbug. His family believe he contracted the infection when he was made to walk barefoot up and down a filthy ward for his physiotherapy. Mr. Drever spent five months in hospital after the superbug invaded his body through the cracked skin on one of his feet. His family plan to sue health chiefs at Colchester General Hospital in Essex. Daughter Sharon said: "I can't accept that my dad was nearly killed by blunders." Dr. Marion Wood, of Essex Rivers Healthcare Trust, said: "He may have become affected in the community, not necessarily when he was here."

Methacillin Resistant S.aureus (MRSA) enters the popular press
I caught MRSA 9 times in the same hospital; NHS Killer Bug Scandal

AN EX-trucker told last night how he lost his right knee after catching the MRSA superbug NINE TIMES in hospital.

Ken Sutton, 60, was repeatedly hit by the deadly infection after an op to rebuild his leg following a motorbike smash.

The bug -which he blames on hygiene blunders -crippled him for life, wrecked his marriage and left him unable to work again.

Ken said last night: "My life is ruined. I had a good job, a loving wife and house of my own and I have lost everything."

"Now I live in a disabled bungalow on Pounds 80 benefits a week and cannot walk properly."

He added: "Having MRSA once is a nightmare -imagine nine times over two years."

"I'd wake up at night screaming in pain. Passers-by in the street would hear and bang on the door."

"I've had a raw deal. To catch it over and over is an appalling indictment of the NHS."
S. aureus and British elections III

We find 80 times danger level of MRSA in hospital

EIGHTY times the danger level of killer superbug MRSA were found at an NHS hospital in a shock investigation by The Sun.

A reporter took samples while working undercover at the North Middlesex Hospital in Edmonton, North London -named a year ago as the joint worst infected in Britain. Bosses vowed then to improve hygiene. But microbiologist Christopher Malyszewicz said of our investigation: "These are the worst results I've seen. It is frightening. The hospital is failing to tackle the problem."

Our reporter took 24 swabs from toilets, soap dispensers, lifts and banisters. An average 40 colony forming units (CFU) were found growing per square centimetre. A reading of 0.53 CFUs is regarded as "the base line" - and is still a risk to patients and visitors.

Mr Malyszewicz, whose company Chemsol tested our samples, said: "In some areas we found 80 times the level which has normally been found, which is unusually high and very dangerous.

Deadly

"Almost every swab you had taken had MRSA present, which is unusual."

"This means MRSA is widespread throughout the hospital and not just in isolated patches."

Deaths from MRSA -Methicillin-resistant Staphylococcus Aureus - have risen 15-fold nationally in the past decade. In 2002, 800 people died, compared with 51 in 1993.
• MRSA kills 5x more Americans than HIV, 60% S. aureus infections resistant, cost of hospital infections $3 \times 10^{10}$.
• Spread via bedrails, wheelchairs, door knobs, stethoscopes, lab coats, blood pressure cuffs (77% contaminated in Fr Hospital).
• Screen, isolate patients can reduce infections 90%. Select your hospital carefully or go to Denmark.
Antibiotics

Most antibiotics are natural products, or derivatives, hence while we prospect the biosphere for novel compounds, the bacteria do likewise for novel defense/signaling mechanisms.*

Strategies:
•pump it out or restrict intake,
•chemically inactivate it,
•modify the target gene by point mutations,
•use completely new genes for targeted function

* A Tomasz, Science 311 342 2006
Antibiotics target many pathways

**Cell wall synthesis**
- Cycloserine
- Vancomycin
- Ristocetine
- Bacitracin
- Cephalosporine

**DNA synthesis**
- Novobiocin
- Nalidixic acid
- Quinolone

**DNA-dependent RNA-polymerase**
- Rifampicin

**Protein synthesis** (30s inhibition)
- Tetracyclin
- Aminoglycosides
- Spectinomycin

**Protein synthesis** (50s inhibition)
- Erythromycin
- Chloramphenicol
- Clindamycin
- Lincomycin

**Cell membrane**
- Colistin
- Polymyxin B

**Folic acid metabolism**
- Trimethoprim
- Sulfonamides

**Cell membrane**
- PABA
Penicillin resistance

Penicillin needed for inhibition (µg/ml)

% strains

1942
1946
1949
1990
Penicillin derivatives, beta-lactams, also induced resistance

1943: Penicillin in therapy

\[
\text{CH}_2\text{CO}-\text{HNCOHNCOOH}
\]

1946-1950: import Penicillinase gene

\[
\text{CH}_2\text{CO}-\text{HNCOHNCOOH}
\]

1960: Methicillin in therapy (resistant to Penicillinase)

\[
\text{CH}_3\text{CO}-\text{HNCOHNCOOH}
\]

1961: import \textit{mecA} gene, does not bind b-lactams

Methicillin resistant strains become global by 1980s

1960: Penicillin resistant staphylococci

become global through plasmid epidemics
### Spread of antibiotic resistance

<table>
<thead>
<tr>
<th></th>
<th>S. aureus ATCC 6538 (1930)</th>
<th>MRSA Brazilian epidemic Clone (1994)</th>
<th>Resistance mechanism acquired (+) adaptive (A)</th>
</tr>
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<tbody>
<tr>
<td>Penicillin</td>
<td>S</td>
<td>R</td>
<td>+ (1945)</td>
</tr>
<tr>
<td>Streptomycin</td>
<td>S</td>
<td>R</td>
<td>+ (1948)</td>
</tr>
<tr>
<td>Tetracycline</td>
<td>S</td>
<td>R</td>
<td>+ (1950)</td>
</tr>
<tr>
<td>Methicillin</td>
<td>S</td>
<td>R</td>
<td>+ (1961) mecA</td>
</tr>
<tr>
<td>Oxacillin</td>
<td>S</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td>Cephalothin</td>
<td>S</td>
<td>R</td>
<td></td>
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<tr>
<td>Cefotaxime</td>
<td>S</td>
<td>R</td>
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</tr>
<tr>
<td>Imipenem</td>
<td>S</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td>Chloramphenicol</td>
<td>S</td>
<td>R</td>
<td>+</td>
</tr>
<tr>
<td>Ciprofloxacin</td>
<td>S</td>
<td>R</td>
<td>A</td>
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<td>Clindamycin</td>
<td>S</td>
<td>R</td>
<td>+</td>
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<td>R</td>
<td>+</td>
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<tr>
<td>Gentamycin</td>
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<td>R</td>
<td>+</td>
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<td>Rifampin</td>
<td>S</td>
<td>R</td>
<td>A</td>
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<td>Vancomycin</td>
<td>S</td>
<td>S</td>
<td>A (1997) VISA</td>
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<tr>
<td>Vancomycin</td>
<td>S</td>
<td>S</td>
<td>+ (2002) vanA</td>
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<td>S</td>
<td>S</td>
<td>+</td>
</tr>
<tr>
<td>Trimeth/Sulfa</td>
<td>S</td>
<td>R</td>
<td>A</td>
</tr>
<tr>
<td>Mupirocine (topical)</td>
<td>S</td>
<td>R</td>
<td>+</td>
</tr>
</tbody>
</table>
Primer on bacterial cell walls
Cell Wall Biosynthesis

Peptidoglycan Biosynthesis

vancomycin binds

Mature Peptidoglycan

Immature Peptidoglycan

vanA

Transpeptidases

Transglycosylases

UDP-MurNAc pentapeptide

Lipid I

Lipid II
Cell wall specific antibiotics

**beta-lactams** (eg methacillin) inhibit the enzymes that link the protein and sugars in the cell wall. Resistance requires foreign gene mecA.

**Vancomycin**: Backup treatment, binds to cell wall precursors
- **Resistance**: Adaptive evolution by several mutations, general ‘stress response’ thickened cell wall, problems dividing, slow growth. Genes involved?
- **Population response**: hetero-resistance strategy, how to prosper in environments +- vanco when resistance imposes growth penalty: spontaneously mutate 1:10^5 bugs, fitness cost 1-10^{-5}. Evolution by design?
Altered cell wall & morphology in VISA strain

wt VISA
Heteroresistance

Amplify bugs good media (no ab), test fraction that grow on plates with incr ab

A: susceptible, B: heteroresistant, C homogeneous resistance
Pick colony from B, re-grow permissive media -> homogeneous resistant to original level and trait genetic.
Resistance -> growth penalty, hetero-resistance good evolutionary strategy.
*meca* gene is associated with large heterologous DNA cassettes

**SCC mec type I**

**SCC mec type III**

**SCC mec type II**

**SCC mec type IV**

Ito et al 2001. AAC  
Grow bugs permissive media, plate on media with increasing antibiotic, count colonies.
Resistance and growth:

- Resistance to vancomycin develops progressively by mutations in multiple genes, what are they?
- Heteroresistance strategy: resistance imposes growth penalty (~2x) thus spontaneously mutate 1:10^5 bugs, fitness cost 1-10^{-5}. What genes involved?
Antibiotic resistance via genome sequencing

1. Intermediate vancomycin resistance multigenetic trait and no obvious ‘resistance’ gene.

2. What mutations are involved in creating heterotypic resistance population analysis curves.

No lack of S.aureus genomes (9 to date), but none close enough (0.01% - 2% point mutation rate + 10’s kb of novel mobile elements)
Compare isolates from one patient undergoing vanco therapy of MRSA.

- Start of infection
- Start of antibiotic therapy
- Death (3 months)

X

JH1

JH9

sequencing
Shotgun sequencing I

genomic DNA

shear DNA

size select

35 ± 3 kbp

6 ± 0.5 kbp

3 ± 0.3 kbp
Shotgun sequencing II

~1000 bp of each end of each fragment is sequenced

read pairs are kept track of
Assembly using Celera assembler

scaffold

contig

contig

Total genome 3 $10^6$ bp
~50 contigs/genome
Use of previously sequenced N315

ancestor

N315

JH1

JH9

1:5000 bp
Multialignment I

N315

size estimate

size estimate

JH1

size estimate

JH9
Read errors and quality values
### Read errors and quality values

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<thead>
<tr>
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<th>N315</th>
<th>GATTCGA</th>
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<td>read 1</td>
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<td>GATTCGA</td>
</tr>
<tr>
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<td>JH1</td>
<td>read 3</td>
<td>GACTCGA</td>
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<td>read 4</td>
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<tr>
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<tr>
<td>read 3</td>
<td></td>
<td>GATTCGA</td>
</tr>
</tbody>
</table>

\[
Q \\
E = 10^{-Q/10}
\]
Bayesian classifier

- column
- JH1 and JH9 reads
- quality values
- phylogeny

Bayesian classifier

- informative
- ND
- uniformative
- no call
- informative
- no ND
Validation of Bayesian classifier by PCR sequencing

- 35 bases mutated
- 6% can’t tell
- 94% no change

3 $10^6$ bp genome
Summary of Assembly and mutations

• 2.9Mb genomes, ~8x coverage, ~60 contigs, ~2% sequence in gaps (30kb plasmid ~50x one contig)
• Changes: N315 vs JH1 about 120kb new sequence in mobile elements, 1:5000 point mutation rate
• Changes: JH1 vs JH9 Call point mutations on 94% of bases (eg 6% = 2% +2% gaps + 2% low coverage) -> 34 total. No larger elements, all gaps consistent with no change
# Temporal order of mutations

<table>
<thead>
<tr>
<th>Isolate</th>
<th>JH1</th>
<th>JH2</th>
<th>JH5</th>
<th>JH6</th>
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<th>JH14</th>
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<td>74</td>
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cccccccccc +c
hypermutable
## Establishment of heart infection

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<tr>
<th>Isolate</th>
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<th>JH2</th>
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</table>
Establishment of heart infection

- JH1: day 1
- JH2: day 63
- JH5: day 74
- JH6: day 79
- JH9: day 86
- JH14: day 90

Legend:
- Blood
- Heart
**Determination of resistance levels**

<table>
<thead>
<tr>
<th>MIC (µg/ml)</th>
<th><strong>Vancomycin</strong></th>
<th><strong>Daptomycin</strong></th>
<th><strong>Rifampin</strong></th>
<th><strong>Oxacillin</strong></th>
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<td>1.0</td>
<td>0.01</td>
<td>0.012</td>
<td>0.75</td>
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<tr>
<th>Isolate</th>
<th>JH1</th>
<th>JH2</th>
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</table>

*Mutations not involved in resistance*

*Mutations discovered from genome*

*Characteristic rif. mutations*

*Primary treatment use*
Potential resistance determinants

<table>
<thead>
<tr>
<th>Vancomycin</th>
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<tbody>
<tr>
<td>SA1702 (in operon with <em>vraSR</em>)</td>
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<tr>
<td><em>agrC</em></td>
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<tr>
<td><em>yycH</em></td>
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<tr>
<td><em>nagB</em></td>
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<thead>
<tr>
<th>Daptomycin</th>
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<tbody>
<tr>
<td><em>yycH</em></td>
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<td><em>rpoC</em></td>
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<th>Rifampin</th>
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<td><em>rpoB</em></td>
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<th>β-lactams</th>
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<td><em>blaR1</em></td>
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</table>

All signaling pathways -> expression of many genes change
Resistance to new Ab, daptomycin, involves similar mutations

vitro evolution created strain with miss-sense mutations in rpoC, rpoB, yycG, (+2 other genes) with reduced daptomycin susceptibility

Friedman L. et al., 2006
Cross resistance

• Daptomycin approved in 2003 for vanco resistant S. aureus (2y after patient died, and not used)
• Common mutations suggest cross resistance between daptomycin and vancomycin.
• JB Patel et al 2006, CDC report documents cross resistance between vancomycin and daptomycin
• Went back and checked our isolates…. 
Jumps in daptomycin resistance coincide with mutations in rpoC and yyc operon

<table>
<thead>
<tr>
<th>Daptomycin MIC (µg/ml)</th>
<th>0.01</th>
<th>0.05</th>
<th>0.05</th>
<th>1.0</th>
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<td>JH14</td>
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- rpoC
- β'- subunit of RNAp
- yycF
- yycG
- yycH
- yycl

E854K

Unknown function. premature Stop
What’s next

Lots more sequencing…

• Is there one path to intermediate vancomycin resistance or many?? Sequence strain collection, study other patients.

• What mutations are responsible for converting heteroresistant MRSA to high homogeneous resistance (untreatable)??

Previous project took 1.5 years
But what about sequencing

1990-2005 costs decrease by 1000x

2005-6, 1 bacteria ~ $10^4, 1 human ~ $2-10\times10^6$, can it go to $1000$
Scanning for mutations via arrays

Genome-Wide Detection of Polymorphisms at Nucleotide Resolution with a Single DNA Microarray

David Gresham,1,2* Douglas M. Ruderfer,1,3 Stephen C. Pratt,1,3 Joseph Schacherer,1,3 Maitrey J. Dunham,1 David Botstein,1,2 Leonid Kruglyak1,3*

A central challenge of genomics is to detect, simply and inexpensively, all differences in sequence among the genomes of individual members of a species. We devised a system to detect all single-nucleotide differences between genomes with the use of data from a single hybridization to a whole-genome DNA microarray. This allowed us to detect a variety of spontaneous single-base pair substitutions, insertions, and deletions, and most (990%) of the ≈30,000 known single-nucleotide polymorphisms between two Saccharomyces cerevisiae strains. We applied this approach to elucidate the genetic basis of phenotypic variants and to identify the small number of single-base pair changes accumulated during experimental evolution of yeast.
Thoughts of Chairman Mao
“Let a hundred flowers bloom, let one hundred schools of thought contend”
Conclusions

• Microevolution + sequencing much more informative than expression arrays (pt mutations in a few pathways vs 200 genes change expression)
• Attractive approach to decipher heteroresistance MRSA, are same genes, operons, pathways, mutated in all isolates??

• Solexa: $400k per machine (300k euros), 3k reagents + 1k overhead per run, multiplex 8 samples, length 30-35 reads (good on poly-N), 99.7% accuracy 1-1.2x10⁹ bp -> $600/bacteria (30x)
Credits

Michael Mwangi (Cornell Physics PhD 2006)

Alex Tomasz (Rockefeller, Shang-Wei Wu, Zhou Yanjiao, Krzysztof Sieradzki, Herminia de Lencastre)

Joint Genome Inst (P. Richards, David Bruce, Eddy Rubin..)

Gene Myers
References

General:
“Antibiotics, Actions, Origins, Resistance”, C Walsh
“Bacterial Pathogenesis..”, Salyers&Whitt

Recent Whole Genome Evolution Papers
M.Olson, PNAS, Pseudomonas in cystic fibrosis
B. Palsson, Nature Genetics Ecoli

WASH YOUR HANDS!
(some of) my encounters with Vinay Ambegaokar

77/78  Postdoc with Vinay at LASSP Cornell
       nonequilibrium superconductivity  cross-country skiing

81    Nordita-Landau Institute meeting in Sweden
       quantum dissipation due to quasiparticle tunneling

84    Workshop on ‘Quantum Dissipation’ at ITP Santa Barbara
       qu. diss. ctd., notions of single-electron effects

86-90s A. v. Humboldt Award for Vinay: visits to Albert Schmid in Karlsruhe
       ballet visit and ice storm
Solid State Qubits and Decoherence

Gerd Schön  
*Karlsruhe*  
http://www.tfp.uni-karlsruhe.de/

**work with:**

Alexander Shnirman  
*Karlsruhe*

Yuriy Makhlin  
*Landau Institute*

Pablo San-Jose  
*Karlsruhe*

Gergely Zarand  
*Budapest*

- Josephson qubits
- Noise and decoherence
- Geometric dephasing of spins in quantum dots
Josephson Charge Qubit

\[ H = E_C \left(2n - \frac{C_g V_g}{e}\right)^2 - E_J \cos(\pi \frac{\Phi_x}{\Phi_0}) \cos \theta \]

2 degrees of freedom
\[ [n, \theta] = -i \text{ charge and phase} \]

2 energy scales \( E_C, E_J \)
charging energy, Josephson coupling

2 control fields: \( V_g \) and \( \Phi_x \)
gate voltage, flux

2 states only, e.g. for \( E_C \gg E_J \)

\[ H = -\frac{1}{2} \Delta E_{ch}(V_g) \sigma_z - \frac{1}{2} E_J(\Phi_x) \sigma_x \]

Two basis/logic states differ by
one Cooper-pair charge \( n \) and \( n+1 \) on island

theory: Caldeira, Leggett (1981)
Shnirman, Makhlin, G.S. (1997+)
expt: Nakamura, Pashkin, Tsai (NEC 1999)
Devoret, Esteve, … (Saclay 2002)

...
Quantum coherent oscillations (Ramsey fringes)

\[ H = -\frac{1}{2} \Delta E_{\text{ch}} (V_g) \sigma_z - \frac{1}{2} E_J (\Phi_x) \sigma_x \]

Nakamura et al. (Nature 99): \( \tau_\phi = 5 \text{ nsec} \)

Vion et al. (Science 02) \( \tau_\phi = 300 \text{ nsec} - 5 \mu\text{sec} \)
Optimum-point strategy for noise reduction

\[ I_{\text{ithier et al.} 2005} \]

\[
\Phi_x/\Phi_0 = -0.3 -0.2 -0.1 0.0
\]

\[
10^{500} 100 10
\]

Coherence times (ns)

Spin echo
Free decay

\[ N_g = C_g \frac{V_g}{2e} \]

\[ |N_g - 1/2| \]
Demonstration of controlled-NOT quantum gates on a pair of superconducting quantum bits

J. H. Plantenberg¹, P. C. de Groot¹, C. J. P. M. Harmans¹ & J. E. Mooij¹
Noise and Decoherence

Sources of noise
- noise from control and measurement circuit, $Z(\omega)$
- background charge fluctuations
- ...

Properties of noise
- spectrum: Ohmic (white), $1/f$, ...
- Gaussian, 2-level systems, ...
- coupling:

$$S_X(\omega) = \frac{1}{2} \int dt \left< \{ X(t), X(0) \}_+ \right> e^{i\omega t}$$

$$\propto \omega \coth \frac{\hbar \omega}{2k_B T}, \quad 1/\omega, ...$$

$$H = -\frac{1}{2} \left( \Delta E \tau_z + X_\parallel \tau_z + X_\perp \tau_x + \lambda X_\parallel^2 \tau_z \right) + H_{\text{bath}}$$

longitudinal – transverse – quadratic (longitudinal) ...
Linear coupling, regular power spectrum

\[ H = -\frac{1}{2}(\Delta E \tau_z + X \cos \eta \tau_z + X \sin \eta \tau_x) + H_{\text{Bath}} \]

Golden rule (Bloch – Redfield) \( \Rightarrow \) exponential decay

(Leggett et al. 87, Weiss 99)

Nyquist noise due to \( R \)

\[ S_{\delta V}(\omega) = \hbar \omega R \ coth \left( \frac{\hbar \omega}{2k_B T} \right) \]

\( \Rightarrow \)

\[ \Gamma_{\text{rel}} \propto (e^2 / h) R \Delta E \]

\[ \Gamma^{\text{\_*}} \propto (e^2 / h) R k_B T \]

Dephasing due to 1/f noise, nonlinear coupling, … ?
longitudinal linear coupling, $1/f$ noise

\[ H = -\frac{1}{2}(\Delta E + X)\tau_z + H_{Bath} \]

\[ S_X(\omega) = \frac{E_{1/f}^2}{|\omega|} \]

Golden rule  \( \Gamma^* = \frac{1}{2} S_X(\omega=0) \) fails for $1/f$ noise, where  \( S_X(\omega) = \to \infty \)

for  \( \omega \to 0 \)

beyond Golden rule:

\[ |\rho_{01}(t)| = \exp\left( -\frac{1}{2} \int \frac{d\omega}{2\pi} S_X(\omega) \frac{\sin^2(\omega t/2)}{(\omega/2)^2} \right) \]

\[ |\rho_{01}(t)| = \exp\left( -\frac{E_{1/f}^2}{2\pi} t^2 \ln |\omega_{ir}t| \right) \]

Non-exponential decay of coherence

Cottet et al. (01)

\[ \Gamma^* \propto E_{1/f} \]
At optimal point: Quadratic longitudinal $1/f$ noise

\[
H = \frac{-1}{2} \left( \Delta E + \lambda X^2 \right) \tau_z \quad S_X(\omega) = \frac{E_{1/f}^2}{|\omega|}
\]

\[
|\rho_{01}(t)| = \left[ 1 + i \frac{2}{\pi} \Gamma_{1/f} t \ln(\omega_{rt}) \right]^{-1/2}
\]

\[\Gamma_{1/f} = \lambda E_{1/f}^2\]  
Shnirman et al. (03,05)

Power law decay

\[e^{-\Gamma t} \quad e^{-\Gamma^2 t^2} \quad 1/|\sqrt{1 + i\Gamma t}|\]

Ithier et al., (2005)

\[\sigma_{\omega}^2/(2\Omega) = 0.025 \text{ Grad/s}\]
Spin qubits: single electrons in quantum dots

Coherent manipulation of coupled electron spins in semiconductor quantum dots
Petta et al. (2005)

Driven coherent oscillations of a single electron spin in a quantum dot
Koppens, …, Vandersypen (2006)
Decoherence of spins in quantum dots

• potentially long phase coherence time!
• fluctuations, e.g., due to piezoelectric phonons
• couple via spin-orbit interaction to spin
• + broken time reversal symmetry
  ⇒ decoherence
• vanishing decoherence for \( \vec{B} = 0 \)

Khaetskii, Nazarov (2001)
Stano, Fabian (2005)

What is spin decoherence at \( \vec{B} = 0 \) ?

We find:
• Nonvanishing decoherence at \( \vec{B} = 0 \)
• due to geometric origin (Berry phase)
• requires two independent sources of fluctuations
• Ohmic fluctuations dominate at low B

San-Jose, Zarand, Shnirman, G.S. (2006)
arXiv:0704.2974
Semiclassical description of spin geometric phase

A: Spin-orbit interaction

\[ H_{SO} = \frac{\hbar}{2} \vec{\sigma} \cdot \vec{B}_{SO} \]

\[ \vec{B}_{SO} = \alpha(p_y, -p_x, 0) + \beta(-p_x, p_y, 0) \]

Rashba
Dresselhaus


B: Semiclassical picture:
electron moves distance \( dr \) in time \( dt \)
\( \Rightarrow \) spin transforms by \( U[dr] \),
independent of \( dt \) (‘geometric’)

\[ U[dr] = \exp \left( \frac{i}{2} \vec{\sigma} \cdot \vec{B}_{SO} \, dt \right) \]

\[ = \exp \left( \frac{i}{2} \vec{\sigma} \cdot \lambda_{SO}^{-1} \cdot dr \right) \]

\[ dr = \frac{\vec{p}}{m^*} \, dt \]

\[ \lambda_{SO}^{-1} = m^* \begin{pmatrix} -\beta & \alpha \\ -\alpha & \beta \end{pmatrix} \]

moving electron + spin orbit interaction \( \leftrightarrow \) ball rolling on surface
Electron with spin in quantum dot

\[ H(t) = \frac{p^2}{2m} + V(\vec{r}) + H_{SO} \]

- Electric field \( \Rightarrow \) displacement
  \[ V(\vec{r}) = \frac{1}{2}m\omega_0^2 r^2 \Rightarrow V(\vec{r}) + e\vec{E}(t)\vec{r} = V\left(\vec{r} + \frac{e\vec{E}(t)}{m\omega_0^2}\right) \]

Spin-orbit coupling
- Eigenstates are spin-textures
- For \( B = 0 \) the states are two-fold degenerate (Kramer’s theorem)
- The lowest doublet is labeled by pseudospin \( \tau \)

\[ \tau = "\downarrow" \quad \tau = "\uparrow" \]

**Slow variation of \( \vec{E}(t) \) \( \Rightarrow \) Adiabatic evolution of lowest doublet**
Spin relaxation and decoherence

Noisy electric field leads to spin diffusion

For weak/slow electric fields $\rightarrow$ effective Hamiltonian

$$H_{\text{eff}} = -\frac{1}{2} \tilde{B}_{\text{eff}} \vec{\tau} + \ldots + \frac{x_0^2 e^2}{2} \left( \dot{E}_\mu E_\nu - E_\mu \dot{E}_\nu \right) \vec{C}_{\mu\nu} \vec{\tau}$$

vanish at zero magnetic field  geometric contribution  requires two orthog. fluctuating fields

Dominant contributions to electric noise:

- Piezo-electric longitudinal phonons: $\rho_{\text{ph}}(\omega) \propto \omega^3$
- Ohmic fluctuations from the leads: $\rho_{\Omega}(\omega) \propto \omega$
  
  (dominant at low energies)
Spin relaxation and decoherence rates

GaAs/AlGaAs lateral quantum dot

$T = 0.1 \text{K}$
dot size = 50 nm
$l_{SO} = 3 \mu \text{m}$
$\alpha/\beta = 1/4$

Saturation at low B

\[
\frac{1}{T_1} \propto S_{\dot{E}_x E_y - E_x \dot{E}_y}(0)
\]

Decay due to random geometric phase due to 2 fluctuating field components

\[
S_{\dot{E}_x E_y - E_x \dot{E}_y}(\omega) \approx 2 \int d\tilde{\omega} \tilde{\omega}^2 S_{E_x} \left( \frac{\omega + \tilde{\omega}}{2} \right) S_{E_y} \left( \frac{\omega - \tilde{\omega}}{2} \right)
\]
Conclusions

• Progress with solid-state qubits
  Josephson junction qubits (exps. 1999+)
  spins in quantum dots (exps. 2005+)

• Crucial: understanding and control of decoherence
  sources and properties of noise,
  coupling mechanisms, decay laws

• Present activities:
  coupling of qubits, coupling of qubit and resonator, …
  pumping, read-out, …

• Spin-offs:
  combine nano-electronics, low-T, and high-frequency technology
  qubits as spectrum analyzer of noise
  quantum optics effects: single-qubit lasing, cooling, …
Entanglement

S. C., Xun Jia, A. Kopp
I. Gruzberg, A. Subramaniam

- Entanglement as a characterization of a ground state wave function.
- A measure of entanglement: Von Neumann entropy
- Entanglement and quantum phase transition.
- 3D Anderson localization.
- Integer quantum Hall plateau transition.
"When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled."

- Schrödinger (Cambridge Philosophical Society, 1935)
Von Neumann entropy as a measure of entanglement

Consider bipartite system and define
\[ \rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}| \]

Then the reduced density matrix
\[ \rho_A = \text{Tr}_B \rho_{AB} \]
is in general a mixture iff the parts are entangled, otherwise it is pure. Then the Von Neumann entropy of A is a measure of entanglement
\[ S = -\text{Tr}\rho_A \ln \rho_A \]

From the Schmidt decomposition theorem it follows that
\[ S = -\text{Tr}\rho_A \ln \rho_A = -\text{Tr}\rho_B \ln \rho_B \]

Schmidt:
\[ |\psi\rangle = \sum_i \sqrt{\mu_i} |i_A\rangle|i_B\rangle, \mu_i \geq 0, \sum_i \mu_i = 1 \]

Not true:
\[ |\psi\rangle = \sum_i \sqrt{\mu_i} |i_A\rangle|i_B\rangle|i_C\rangle \]
Quantum critical points as highly entangled states

\[ S_{\text{thermal}} = - \left( \beta \frac{\partial}{\partial \beta} - 1 \right) \ln \text{Tr} \ e^{-\beta H} \]

\[ S_{\text{vNeumann}} = - \left( n \frac{\partial}{\partial n} - 1 \right) \ln \text{Tr} \ \rho_A^n |_{n \to 1^+} \]

\[ \text{Tr} \ \rho_A^n = \sum_i \mu_i^n < \sum_i \mu_i = 1, \ \mu_i : [0, 1) \]

The l.h.s. is absolutely and uniformly convergent and therefore analytic for all \( \Re n > 1 \). The non-analyticity must be in \( \rho_A \)

- Conjecture: the Von Neumann entropy is non-analytic at a quantum phase transitions, in particular at a quantum critical point. It is otherwise analytic.

- There are cases where the ground state energy is analytic but the Von Neumann entropy is non-analytic at the quantum phase transition. This is not the traditional picture of a quantum phase transition, for example, Ising model in a transverse field.
Quantum phase transition and quantum critical point

One of the simplest examples is

$$|0⟩ = \prod_i (|↑⟩ + |↓⟩ \sqrt{2})$$

1. When $\lambda \rightarrow 0$ the ground state is the direct product state of qubits---not entangled

$$|0⟩ = \prod_i (|↑⟩ + |↓⟩ \sqrt{2})$$

Note that it is not the same as the state

$$|0⟩_S = \frac{1}{\sqrt{2}} \left( \prod_i |↑⟩ + \prod_i |↓⟩ \right)$$

2. When $\lambda \rightarrow \infty$ the ground state is the unentangled direct product

$$|0⟩ = \prod_i |↑⟩$$
What happens when $\lambda$ increases from 0?
More and more qubits get entangled, but the symmetry remains unbroken until $\lambda = 1$

At $\lambda = 1$, there is a quantum phase transition to a broken symmetry state-- $\lambda = 1$ is a quantum critical point.

The second derivative of the ground state energy density is logarithmically singular $\lambda = 1$

$$e_0 = -\frac{2J}{\pi}(1 + \lambda)E\left(\frac{2\sqrt{\lambda}}{1 + \lambda}\right)$$

It is also easy to compute the single site Von Neumann entropy, where the subsystem A is the site and B is the rest.

$$\rho_A = \frac{1}{2} \begin{pmatrix} 1 + \langle \sigma_i^z \rangle & \langle \sigma_i^x \rangle \\ \langle \sigma_i^x \rangle & 1 - \langle \sigma_i^z \rangle \end{pmatrix}$$

$$\lim_{\lambda \to 1^-} \frac{\partial S}{\partial \lambda} = -\frac{1}{2\pi} \ln \left(\frac{\pi + 2}{\pi - 2}\right) \ln |\lambda - 1|$$

$$\lim_{\lambda \to 1^+} \frac{\partial S}{\partial \lambda} = -\frac{\pi}{2^{19/4}} \ln \left(\frac{\pi + 2}{\pi - 2}\right)(\lambda - 1)^{-3/4}$$
Anderson localization

\[ H = \sum_{n} \varepsilon_{n} c_{n}^\dagger c_{n} - t \sum_{\langle m,n \rangle} (c_{n}^\dagger c_{m} + h.c.) \]

\[ -W/2 \leq \varepsilon_{n} \leq W/2 \]

All states are localized for dimensions \( D \leq 2 \) but there is a metal-insulator transition for \( D = 3 \) for a critical \( W/t \)

A theorem due to Thouless

Impurity averaged ground state energy is analytic.

\[ \overline{G}_{mn} = \int G_{mn}(E, \{\varepsilon_{n}\})d\mu\{\varepsilon_{n}\} \]

The density of states must be analytic in the real energy range

\[ -\frac{W}{2} + Zt \leq E \leq \frac{W}{2} - Zt \]
\[ \rho_n = z_n (|1\rangle\langle 1|)_n + (1 - z_n) (|0\rangle\langle 0|)_n, \quad z_n = |\psi_n^{\alpha}|^2 \]

\[ S_{Vn} = -z_n \ln z_n - (1 - z_n) \ln (1 - z_n) \]

\[ \overline{S_V} = \frac{1}{MN} \sum_{\alpha,n} S_{Vn}^{\alpha} \]

From multifractal analysis

\[ L^d \overline{S_V} = C + (\ln L) L^{-1/\nu} f(L^{1/\nu} w) \]

\[ \overline{S_V}(E = 0, w) = k(w) \ln L + C(w) \]

\[ k(w) \rightarrow d, \text{ metallic} \]

\[ \rightarrow 0, \text{ insulating} \]

\[ \rightarrow \alpha_1 = 2, w = 0 \]

\[ L = N^{1/3} \]

\[ w = |W - W_c|/W_c \]
$W_c = 16.5$

$v = 1.48$

$C = 12.95$

$(sL^d - C)L^{1/v}/\ln(L)$
\[ S_L = -\text{Tr} \rho_A (\rho_A - 1) = 1 - \text{Tr} \rho_A^2 \]

\[ P^{(2)} = \frac{1}{\mathcal{A}} \sum_{i,\alpha} |\psi_i^\alpha|^4 = 1 - S_L / 2 \quad \text{(participation ratio)} \]

\[ P^{(2)} = L^{-x} g_\pm (L^{1/\nu} w) \quad x = 1.4 \]
Integer Quantum Hall effect

\[ H = \sum_{n,k} |n, k\rangle\langle n, k| \left( n + \frac{1}{2} \right) \omega_c + \sum_{n,k} \sum_{n',k'} |n, k\rangle\langle n, k| V |n', k'\rangle\langle n', k'| \]

Projected to the lowest Landau level: \[ \langle 0, k|V|0, k' \rangle \]

\[ L = L_x = L_y = \sqrt{2\pi M l_B} \quad l_B = (\hbar/eB)^{1/2} \]

\[ k = 2m\pi/L \quad m = 0, 1, \ldots, N_k - 1 \quad N_k = BL^2/\phi_0 = 2M^2 \]

\[ |\psi\rangle = \sum_k \alpha_k |0, k\rangle \]

\[ \psi(x, y) = \langle x, y|\psi\rangle = \sum_k \alpha_k \psi_{0,k}(x, y) \]
Plateau transition

\[ A : (\pi/2)(l_B)^2 \]

\[ z = \int_{(x,y) \in A} |\psi(x, y)|^2 dxdy \]

\[ S = -z \ln z - (1 - z) \ln(1 - z) \]

\[ E \] is measured in units of

\[ \Gamma = \frac{2V_0}{\sqrt{2\pi l_B}} \]

\[ \overline{V(r)V(r')} = V_0^2 \delta(r - r') \]

\[ \nu \approx \frac{7}{3} \]
Conclusions

• Entanglement entropy seems to correctly capture quantum phase transitions and may provide additional insight.

• Entanglement entropy may be an effective way of detecting hidden orders in interesting many body ground state.
Symposium in honor of Vinay Ambegaokar
Ithaca, June 16, 2007

„Supervision by benevolent neglect...“

„What, if anything, have you done lately?“

„I have turned the pages of your draft. Quite frankly, I found it to be unreadable...“

„He emphasized at least 25 times that his results were exact, but when asked, had to admit that his model was based on an approximation!“

„Follow the phenomena!“
Mesoscopic to Universal Crossover of Transmission Phase of Multi-Level Quantum Dots

Aharonov-Bohm interferometry: experimental setup

Yacoby et al., PRL 74, 4047 (1995)

$$|t_{SD}|^2 = \sum_{\sigma} \left| t_{u\sigma} e^{i2\pi\Phi/\Phi_0} + t_{d\sigma} \right|^2$$

$$= \sum_{\sigma} \left[ |t_{u\sigma}|^2 + |t_{d\sigma}|^2 + 2|t_{u\sigma}| |t_{d\sigma}| \cos(2\pi\Phi/\Phi_0 + \phi_{u\sigma} - \phi_{d\sigma}) \right]$$
**Expected Phase Evolution for Resonant Tunnelling**

Two terms can cancel completely. Then transmission amplitude = 0, transmission phase jumps by $\pi$!

Simple explanation for phase lapse:
Sign change of transmission amplitude, due to „destructive interference“ between two partially overlapping levels:

\[ t_{d\sigma} \propto \frac{t_{i}^{L} t_{i}^{R} t_{j}^{L} t_{j}^{R}}{(\varepsilon - \varepsilon_{i}) + i\Gamma_{i} + (\varepsilon - \varepsilon_{j}) + i\Gamma_{j}} \]

Abrupt phase lapse between the peaks, with width $< G_{e} , < k_{B}T$
Moty Heiblum’s phase lapse puzzle…

Prediction of model of noninteracting electrons:

\[ \alpha / \pi \]

\[ \theta \]

\[ + \quad - \quad + \quad - \quad - \quad + \quad - \quad - \quad + \quad - \quad - \]

\[ \text{signs: } s_j = \text{sgn}(t_j^L t_j^R t_{j+1}^L t_{j+1}^R) = \pm \]


Results for many-electron quantum dots:

- Phase lapse occurs in every conductance valley!
- “Universal phase lapse behavior”
- Sample-specific details not matter!??
- Triangular conductance peak shapes!
Some of theory papers addressing phase lapse puzzle...


Moty Heiblum: “No single acceptable explanation !”

31. V. Kashcheyevs, A. Schiller, A. Aharony, O. Entin-Wohlman, PRB 75, 115313 (2007) [2 levels, unified picture]

Moty Heiblum: “theoretical picture seems to be converging...”
Few-electron AB interferometry


Crossover from ‘universal’ to ‘mesoscopic’ behavior
Summary of experimental findings:

(1) For occupation 1 to about 10, the phase evolution is highly sensitive to the dot’s configuration and occupation.

(2) For occupation higher than about 14, phase evolution is universal-like, with phase ranging only between 0 and $\pi$, and is independent of the dot’s configuration and occupation.

Speculation on possible mechanism:

„An outstanding feature of larger quantum dots is the smaller level spacing, which might lead to levels overlapping. This will favour models that invoke transmission mediated through more than one quantum state.“ — Avinun-Kalish et al., Nature 2005
Multi-level Dot Model (spinless electrons)

\[ H_{\text{lead}} = \sum_k \sum_\mu \varepsilon_{k\mu} c_{k\mu}^\dagger c_{k\mu} \]

\[ H_{\text{dot}} = \sum_{ij} \left( \varepsilon_j - V_g \right) \delta_{ij} d_i^\dagger d_j + \frac{1}{2} U \sum_{i \neq j} (\tilde{n}_i - \frac{1}{2})(\tilde{n}_j - \frac{1}{2}) \]
\[ \delta = \varepsilon_{j+1} - \varepsilon_j \]

\[ H_{\text{lead-dot}} = \sum_k \left[ c_{kL}^\dagger \sum_j t_j^L d_j + c_{kR}^\dagger \sum_j t_j^R d_j \right] \]

Partial widths: \( \Gamma_j^\mu = \pi \rho |t_j^\mu|^2 \), Average: \( \Gamma = \frac{1}{N} \sum_{j\mu} \Gamma_j^\mu \)

Signs of \( t_j^\mu \): \( s_j = \text{sgn}(t_j^L t_j^R t_{j+1}^L t_{j+1}^R) = \pm \)

Collective notation: \( \gamma = \{ \Gamma_1^L, \Gamma_1^R, \Gamma_2^L, \ldots \}, \quad \sigma = \{ s_1, s_2, \ldots \} \)
Transmission Amplitude

derived from results of: Bruder, Fazio, Schoeller PRL, 76, 114 (1996)

\[ T_{LR} = 2\pi \rho \sum_{ij} t_i^L G_{ij}^{0R} t_j^R = |T| e^{i\alpha} \]

\[ \overline{G}_{ij}^{0R} = \int d\varepsilon \left[ -\partial _{\varepsilon} f(\varepsilon) \right] G_{ij}^{0R}(\varepsilon) \delta(\varepsilon) \text{ for } T = 0. \]

Assumptions:
1. phase-coherent transport
2. only one mode carries current
3. no multiple traversals of ring
4. dot stays in equilibrium

Numerical Renormalization Group (NRG)
[iterative diagonalization of Hamiltonian]

Functional Renormalization Group (fRG)
[RG-improved perturbation theory in U]
NRG: $\delta/\Gamma = 20$  (mesoscopic)

<table>
<thead>
<tr>
<th>$N=4$</th>
<th>$U/\Gamma=24$, $\sigma=--+$, $\gamma/\Gamma={0.16, 0.24, 0.32, 0.48, 0.72, 0.48, 0.68, 0.92}$, $T/\Gamma=0.004$</th>
</tr>
</thead>
</table>

Relative sign of couplings:
NRG: $\delta/\Gamma = 16$ (crossover)

$N=4$, $U/\Gamma=24$, $\sigma=---+$, $\gamma/\Gamma=\{0.16, 0.24, 0.32, 0.48, 0.72, 0.48, 0.68, 0.92\}$, $T/\Gamma=0.004$
NRG: $\delta/\Gamma = 12$ (crossover)
NRG: $\delta/\Gamma = 4$ (crossover)
NRG: $\delta/\Gamma = 0.4$ (universal behavior of phase)

For $\delta < \Gamma$, phase lapses occur in each conductance valley!
**fRG at zero temperature**

Using fRG, we found this to happen generically, for many different parameter sets.

For $\delta < \Gamma$, phase lapses occur where we want them to be!
For $T=0$, NRG and fRG agree qualitatively very well, except for $\delta < \Gamma, U >> \delta$.

For $T\neq 0$, sharp features are smoothed out, results similar to exp.!!
Qualitatively, experimental trends are reproduced!

But the question remains: what is the mechanism at work here? fRG has the answers!

(for $T>\delta$, the $\delta$-dependence is weak)
**fRG: RG-enhanced perturbation theory in U**

$U = 0$: integrate out leads:

$$ G_0^R(0)_{ij}^{-1} = -h_{ij}^0 + i \Delta_{ij} $$

eigenvalues: $-\varepsilon_j + i\Gamma_j$

effective level positions and widths of bare single-particle model.

$U \neq 0$: RG-improved pert. theory in $U$:

$$ G_R(0)_{ij}^{-1} = -h_{ij}^0 + \Sigma_{ij} + i\Delta_{ij} $$

eigenvalues: $-\tilde{\varepsilon}_j + i\tilde{\Gamma}_j$

effective level positions and widths of renormalized single-particle model.
This leads to Coulomb blockade resonances in the transmission.

For $\delta >> \Gamma$, level positions vary roughly linearly with $V_g$.

Each time one of the narrow levels crosses the chemical potential, its occupation increases from 0 to 1.

This leads to Coulomb blockade resonances in the transmission.

The phase climbs by $\pi$ across each resonance; its behavior between resonances depends on signs of matrix elements.
Transmission amplitude has form leading to destructive quantum interference between broad and the narrow level: "Fano effect" !!

For time-reversal symmetry, this always causes transmission zero ("Fano antiresonance") and phase lapse.

For $\delta < \Gamma$, one effective level is much broader than others.

Its energy hovers near chemical potential of leads, while its occupancy stays near $\frac{1}{2}$.

Each time one of the narrow levels crosses the chemical potential of leads it also crosses the broad level.

Transmission amplitude has form

$$t_{\text{wide}} + \frac{t_{\text{narrow}}}{\varepsilon + i\tilde{\Gamma}_{\text{narrow}}}$$

leading to destructive quantum interference between broad and the narrow level: "Fano effect" !!

For time-reversal symmetry, this always causes transmission zero ("Fano antiresonance") and phase lapse.
Our scenario differs from theirs, which assumes $U >> \Gamma_{\text{wide}}$. 
Origin of broad level (Dicke effect)

\[ G^R(0)^{-1}_{ij} = \left( -h^0_{ij} + \sum_i \Delta_{ij} + i \pi \rho \sum_\alpha t^\alpha_i t^\alpha_j \right) \]

Eigenvalues: \(-\tilde{\varepsilon}_j + i \tilde{\Gamma}_j\)

Rank two matrix, has 2 nonzero eigenvalues. (larger one dominates).

Dominates if \(\delta > \Gamma\)

Dominates if \(\delta < \Gamma\)

For \(\delta \ll \Gamma\), broad level emerges automatically

(for large \(N\), \(\tilde{\Gamma}_{\text{max}} \sim N\))

Renormalized level widths are essentially independent of \(U\) (!)

\[ N=4, \frac{V_g}{\Gamma}=0.0, \sigma=\{+-\} \]

50,000 RUNS, \(b/\Gamma=0.2\)

\[ <\tilde{\Gamma}> / \Gamma \]

\[ \delta / \Gamma \]
Noninteracting model with single wide, hovering level


**Assumptions:**

No interactions: $U=0$

Many narrow levels, uniform spacing $\delta$

One wide, hovering level, position: $\epsilon_{\text{wide}}=0$, width: $\gamma \gg \Gamma_{\text{narrow}}$

**Meso:** $\gamma=0, \Gamma=0.25\delta$

Universal Asymmetric $\gamma=5$, $V_L=4V_R=0.1\,\text{V}$

```
|t|^2
-Vg/\delta

\arg(t)/\pi
```
Conclusions

- Broad level emerges naturally once $\delta < \Gamma$
- Broad effective level produces Fano-antiresonances with narrow levels.
- Results are in qualitative agreement with experiment.

Open issues

- Study role of spin for N=3, 4 levels... (presumably not relevant in regime $\delta < \Gamma$)
- May we use bare parameters with $\delta < \Gamma$?
  Conventional wisdom says this is not generic.
- Understand better which conditions produce broad levels (geometry-dependent?)
- Find out by doing detailed modelling!

work in progress...
Thank you for your attention!
Scenario of Silvestrov & Imry


Assumed one broad level (backed up by model calculations for chaotic dots)
Assumed $U \gg \Gamma_{\text{wide}}$

Showed that this leads to succession of (strong) occupation inversions:
- wide level fills up, gets emptied into narrow level, fills up again, etc.

Pointed out that a phase lapse occurs at each occupation inversion,
- implying universal succession of

How does our work differ from theirs?
- we do not assume a wide level: it appears generically for $\delta < \Gamma$ (Dicke effect)
- condition $U \gg \Gamma_{\text{wide}}$ is not needed to find universal phase lapses
- whether (strong) occupation inversion occurs or not is irrelevant (it is a side effect).
- essential physics is due to Fano interference of broad with narrow level
Role of spin?

Presumably not relevant in regime $\delta < \Gamma$, where no Kondo physics occurs.
**fRG: RG-enhanced perturbation theory in U**

\[ U = 0: \text{integrate out leads} \quad U \neq 0: \text{RG-improved pert. theory} \]

\[
G^{0R}(0)_{ij}^{-1} = -h_{ij}^0 + i \Delta_{ij} \quad \text{RG} \quad G^{R}(0)_{ij}^{-1} = -h_{ij}^0 + \sum_{ij} + i\Delta_{ij}
\]

Eigenvalues: \(-\varepsilon_j + i\Gamma_j\)

Effective level positions and widths of renormalized single-particle model.

RG introduces infrared cutoff \(\Lambda\), and reduces it to zero in infinitesimal steps

\[
G^\Lambda(i\omega_n) = \chi^\Lambda(\omega_n)G_0(i\omega_n)
\]

\[
G^\Lambda = \left[ [G^{0,\Lambda}]^{-1} - \Sigma^\Lambda \right]^{-1}
\]

\[
G^\Lambda \partial_\Lambda [G^{0,\Lambda}]^{-1} G^\Lambda
\]

\(\Lambda_{\text{final}} = 0\Lambda - \pi T\)  \(\Lambda + \pi T\)  \(\Lambda_{\text{initial}} = \infty\)
fRG: emergence of broad level

$N=4, \ U/\Gamma=4, \ \sigma=\{\ldots\}, \ \gamma/\Gamma=\{0.266, 0.533, 0.4, 0.8, 0.133, 0.266, 0.533, 1.07\}$

$\delta/\Gamma=16$
fRG: emergence of broad level

$N=4$, $U/\Gamma=4$, $\sigma=\{---\}$, $\gamma/\Gamma=\{.266,.533,.4,.8,.133,.266,.533,1.07\}$

$\delta/\Gamma=12$

![Graph showing various plots with $V_g / \Gamma$ on the x-axis and other variables on the y-axis.](image)
fRG: emergence of broad level

$$N=4, \frac{U}{\Gamma}=4, \sigma=\{\ldots\}, \frac{\gamma}{\Gamma}=\{0.266, 0.533, 0.4, 0.8, 0.133, 0.266, 0.533, 1.07\}$$

$$\frac{\delta}{\Gamma}=4$$
fRG: emergence of broad level

\[ N=4, U/\Gamma=4, \sigma=\{\ldots\}, \gamma/\Gamma=\{.266, .533, .4, .8, .133, .266, .533, 1.07\} \]

\[ \delta/\Gamma=2 \]
fRG: emergence of broad level destructive interference between two levels

\[ \mathbf{N}=4, \ U/\Gamma=4, \ \sigma=\{\ldots\}, \ \gamma/\Gamma=\{0.266, 0.533, 0.4, 0.8, 0.133, 0.266, 0.533, 1.07\} \]

\[ \delta/\Gamma=1 \]

destructive interference between two levels
fRG: emergence of broad level

\[ N = 4, \quad U/\Gamma = 4, \quad \sigma = \{\ldots\}, \quad \gamma/\Gamma = \{0.266, 0.533, 0.4, 0.8, 0.133, 0.266, 0.533, 1.07\} \]

\[ \delta/\Gamma = 0.85 \]

(a) \[ \varepsilon_j/\Gamma \]

(b) \[ \delta_j \]

(c) \[ \alpha/4\pi, \quad \tilde{T_j}/N \Gamma \]

\[ \tilde{T_j} \]

\[ \alpha \]

\[ |T| \]

\[ V_g/\Gamma \]

hovering level

hovering level is broad

destructive interference between two levels
fRG: emergence of broad level

\[ N=4 \quad U/\Gamma=4, \quad \sigma=\{\ldots\}, \quad \gamma/\Gamma=\{0.266, 0.533, 0.4, 0.8, 1.33, 0.266, 0.533, 1.07\} \]

\[ \delta/\Gamma=0.7 \]

Hovering level

Hovering level is broad

destructive interference between two levels
fRG: emergence of broad level

N=4 \ U/\Gamma=4, \ \sigma=\{---\}, \ \gamma/\Gamma=\{.266,.533,.4,.8,.133,.266,.533,1.07\}

\(\delta/\Gamma=0.4\)

(a) Hovering level

(b) Hovering level is broad

destructive interference between two levels
fRG: emergence of broad level

N=4, U/Γ=4, σ={---}, γ/Γ={.266,.533,.4,.8,.133,.266,.533,1.07}

δ/Γ=0.2

hovering level

hovering level is broad

destructive interference between two levels
Charge detection by point contact
NRG: numerical diagonalization method

\[
H = H_{\text{dot}} + \sum_{\mu,j} \int_{-D}^{D} d\epsilon \; t_j^\mu \left( a_{\epsilon \mu}^{\dagger} d_j + d_j^{\dagger} a_{\epsilon \mu} \right) + \sum_{\mu} \int_{-D}^{D} d\epsilon \; a_{\epsilon \mu}^{\dagger} a_{\epsilon \mu}
\]

can be mapped to quantum chain with exponentially decreasing coupling

\[
H = H_{\text{dot}} + \sum_{\mu,j} t_j^\mu \left( f_{0,\mu}^{\dagger} d_j + d_j^{\dagger} f_{0,\mu} \right) + \sum_{\mu} \sum_{n=0}^{\infty} \Lambda^{-\frac{n}{2}} \left( f_{n,\mu}^{\dagger} f_{n+1,\mu} + f_{n+1,\mu}^{\dagger} f_{n,\mu} \right)
\]
Number parity in superconductors and the $\nu=5/2$ fractional quantized Hall state

by Bertrand I. Halperin

Harvard University

Talk in honor of Vinay Ambegoakar on the occasion of his retirement. Ithaca, June 16, 2007
Number Parity in Superconductors

Dependence of free energy, other quantities, on whether the number of electrons in a small sample is even or odd

**BCS superconductivity with fixed number parity**

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(Received 28 February 1994)

**Parity Fluctuations between Coulomb Blockaded Superconducting Islands**

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(Received 14 October 1994)
Moore and Read (1991) proposed a novel trial wave function as a model to explain the observed even-denominator quantized Hall state at filling fraction 5/2. Elementary charged excitations have charge $\pm e/4$ and obey “non-abelian statistics”.

State is related mathematically to a superconductor with $p_x + ip_y$ pairing.

The ground state energies of these states have, in common a peculiar dependence on number parity. This is important for a proposed experiment to verify the Moore-Read conjecture for the $\nu = 5/2$ state.
Outline

What do we mean by Non-Abelian statistics? How do they work in the Moore-Read “Pfaffian” state?

Description of the Pfaffian state as a p-wave BCS superconductor of “composite fermions”. $\Delta \sim (p_x + ip_y)$.

Properties of a $p_x + ip_y$ superconductor.

Our proposed experiments and their predicted peculiar results for the Moore-Read state. Relation to number parity.
Non-abelian statistics for Moore-Read 5/2 state

Consider a system containing $2N$ localized quasiparticles, far from each other and far from boundaries. Then there exist $M=2^{N-1}$ orthogonal degenerate ground states, which cannot be distinguished from each other by any local measurement.

Moving various quasiparticles around each other and returning them to their original positions, or interchanging quasiparticles, can lead to a nontrivial unitary transformation of the ground states, which depends on the order in which the winding is performed. (Unitary matrix depends on the topology of the braiding of the world lines of the quasiparticles. Matrices form a representation of the braid group).

If two quasiparticles come close together, degeneracy is broken; but energy splittings fall off exponentially with separation.
Topological quantum computation

Non-abelian quasiparticles may be useful for “topological quantum computation”.


Manipulation of qubits would be carried out by moving quasiparticles around each other, not bringing them close together. Advantage: exponentially long decoherence times.

Caveat: Moore-Read state is not rich enough for general topological quantum computation.

More complicated non-abelian states have been proposed, which would allow universal quantum computation. (E.g., Read-Rezayi $k=3$ state; may be realized at $\nu = 12/5$.)
Quasiparticles in the quantized Hall state correspond to vortices in the superconductor.

For a BCS superconductor with pairing function $\Delta \propto (p_x + ip_y)$: Vortices have zero energy states. If there are $2N$ vortices present, there are $M = 2^{N-1}$ degenerate ground states.

Moving vortices around each other generates a unitary transformation on these states similar to that for quasiparticles in the Moore-Read state.

*Effects of vortex motion on zero energy states elucidated by Ivanov (PRL 2001); and Stern, von Oppen and Mariani (PRB 2004), using Bogoliubov-de Gennes equations for superconductor.
Zero-energy modes

Specifically, in a $p_x+ip_y$ superconductor, an isolated vortex, at point $R_i$, has a zero energy mode, with Majorana fermion operator $\gamma_i$:

\[
\gamma_i = \gamma_i^\dagger, \quad \gamma_i^2 = 1, \quad \{\gamma_i, \gamma_j\} = 2\delta_{ij}
\]

To form ordinary fermion creation or annihilation operator: need pair of vortices: e.g.

\[
c_{12} = (\gamma_1 + i\gamma_2) / 2, \quad c_{12}^\dagger = (\gamma_1 - i\gamma_2) / 2
\]

obey usual fermion commutations rules

$N_{12} = c_{12}^\dagger c_{12}$ has eigenvalues $= 0, 1$. $[N_{12}, N_{34}] = 0$, etc.

Constraint: Number of occupied pairs $= N_{\text{electrons}} \pmod{2}$.

$\Rightarrow$ 2N vortices gives $2^{N-1}$ independent states
Explicit relation between Majorana operator and electron operators

\[ \gamma_i = \int d\mathbf{r} \left[ u(\mathbf{r}) \psi(\mathbf{r}) + v(\mathbf{r}) \psi^\dagger(\mathbf{r}) \right] \]

with \( v(\mathbf{r}) = u^*(\mathbf{r}) \), localized near vortex.

If vortices are far apart, so there is no overlap between the wave functions of their zero-energy states, then these states must have precisely zero energy.

This relates to the fact that solutions of the BdG equations must occur in pairs with \( E_1 = -E_2 \).
Boundary states

Finite sample with an odd number of elementary vortices will have a zero energy state at the boundary. Form pair between boundary state and one of the vortices. Again have constraint: Number of occupied pairs = \( N_{\text{electrons}} \) (mod 2).

Generally, edge of superconductor has a series of low-energy fermion modes, with energies \( E_m = m \left( \frac{\pi v}{L} \right) \),

\[ m = 0, \pm 1, \pm 2, \ldots \text{ if number of vortices is odd}, \]

\[ m = \pm 1/2, \pm 3/2, \ldots \text{ if number of vortices is even}. \]

Get even-odd alternation in energy to add an electron to the system, if number of vortices is even, not if number of vortices is odd. Alternation energy \( \sim \frac{v}{L} \); goes to 0, for \( L \rightarrow \infty \).
Contrast to s-wave superconductor

s-wave superconductor has no low-energy fermion states at boundary.

Energy to add an electron has an even-odd alternation independent of whether there are an even or odd number of vortices present, and independent of the perimeter of the sample.

True also for gapless d-wave superconductor, $\Delta \propto (p_x + ip_y)^2$, or for a Bose condensate of tightly bound p-wave pairs.
Braiding properties of vortices

Vortices at points $R_1$ $R_2$ $R_3$ $R_4$
Braiding properties

Vortices at points $R_1$, $R_2$, $R_3$, $R_4$

Move vortex 2 around vortex 3. Gives unitary transformation $\sim \gamma_2 \gamma_3$.

Changes $N_{12} \rightarrow (1-N_{12})$, $N_{34} \rightarrow (1-N_{34})$. 
Braiding properties

Vortices at points $R_1 \ R_2 \ R_3 \ R_4$

Move vortex 2 around 3 and 4. Gives unitary transformation
$\sim \gamma_2 \gamma_4 \gamma_2 \gamma_3 = \gamma_3 \gamma_4$ : leaves $N_{12}$ and $N_{34}$ unchanged.

Since vortices are indistinguishable, get other unitary transformations by simply interchanging positions of two vortices.

Order of interchanges matter: The unitary transformations do not commute.
Connection between Moore-Read State and $p_x+ip_y$ Superconductor

Moore-Read state is a $p_x+ip_y$ superconductor of “composite fermions”

Connection can be understood through the Fermion-Chern-Simons transformation. Apply a unitary transformation to convert interacting electron system into a set of fermions interacting with a Chern-Simons gauge field (long-range momentum-dependent interaction). Two CS flux quanta are attached to each fermion. In mean field approximation, replace CS field by average field, $\langle b \rangle = 4 \pi n_e$ (in units where $e=1$).

$\nu=5/2$: one filled Landau level with both spin states occupied + $\nu=1/2$ for electrons in the second Landau level. Assume spin polarized.
If the electron filling fraction is $\nu=1/2$

2 flux quanta of actual magnetic field per electron.

Effective magnetic field = $B - \langle b \rangle = 0$.

Mean-field ground state = filled Fermi Sea \[ k_F = (4\pi n_e)^{1/2} \]

If this is correct, then there is no energy gap, no QHE.

Should be able to calculate all properties of $\nu=1/2$ state using perturbation theory, starting from the mean field state.

Perturbations include effects of $v(\mathbf{r}_i-\mathbf{r}_j)$ and fluctuations in the Chern Simons field \[ \Delta b_i \equiv b_i - \langle b \rangle. \]
Depending on the short-distance interactions between fermions the Fermi surface may be unstable, e.g., to **formation of p-wave superconductivity**

If a superconducting energy gap forms at the Fermi surface, then state is stabilized at precisely $\nu = 1/2$. Deviations in filling fraction $\Rightarrow B_{eff} \neq 0 \Rightarrow$ requires vortices, costs finite energy.

Get plateau in Hall conductance at $\nu = 1/2$ : fractional quantized Hall state.

Apparently: Superconductivity does not occur for electrons in the lowest Landau level ($\nu = 1/2$) but does occur for electrons in the second Landau level.
Moore-Read quasiparticle \(\leftrightarrow\) vortex in superconductor

By Meissner effect, vortex must bind 1/2 quantum of magnetic flux to have finite energy. With a Chern-Simons gauge field, the source of magnetic flux is charge, rather than current.

1/2 quantum of Chern-Simons flux requires \(1/4\) electric charge.
What is the evidence that the $\nu = 5/2$ Quantized Hall State is indeed of the Moore-Read type?

Evidence comes primarily from numerical calculations on finite systems. (Morf & collaborators, 2002, 2003; Das Sarma et al. 2004).

Using electron-electron interactions appropriate for electrons in the second Landau level, with parameters appropriate to GaAs samples, find a spin-polarized ground state, which seems to have an energy gap, and which has good overlap with Pfaffian wave function. Relatively small changes in parameters can lead to other ground states, which are not quantized Hall states. (As is found experimentally for samples in large in-plane magnetic field.)
Proposed experiments

Moving one quasiparticle around another can be done in principle by means of gates which couple electrostatically to the charge of the quasiparticles; but we are far from being able to accomplish this technologically.

We seek other experiments to examine the $\nu=5/2$ state to see if it is of the Moore-Read type.

*Measurements of the quasiparticle charge. (e.g. using SETs, as in studies of $\nu=1/3$ by Yacoby et al.) Moore-Read quasiparticles have charge $e/4$.

*Measurements of spin polarization. Moore-Read has complete polarization in second Landau Level.

*Interference-type experiments related to non-abelian statistics.
Proposed Interference Experiments

Discussed by: Ady Stern and B. I. Halperin

Other theoretical papers discussing interference experiments with non-abelian quasiparticles include:

Das Sarma, Freedman and Nayak, (PRL 2005)

Bonderson, Kitaev, and Shtengel, (PRL 2006)

Fradkin et al., Nucl Phys B 1998

Bonderson, Shtengel and Slingerland, cond-mat/0601242: Discuss consequences for Read-Rezayi parafermion states, possibly applicable to ν=12/5.
Fix gate voltage at point contacts. Vary area A by varying voltage on side gate. Measure resistance $V_{12}/I$. Expect oscillations in the resistance as a function of A.

$\nu = 5/2$

$t_1$

$t_2$

$\nu = 5/2$

$I$

$+ = \text{quasihole}$

Side Gate
If \( v = 1/2 \) state is non-abelian Pfaffian state: the period of resistance oscillations (for varying A) should depend on whether quasihole-number is even or odd.
Coulomb blockade regime, $V_{12}$ is large, except on resonance, when $E_N = E_{N+1}$
If the number of quasiholes is even, $E_N$ has an even-odd alternation in the electron number $N$.

The period for oscillations in $V_{12}$ is then $\Delta A = 4\Phi_0/B$, corresponding to the addition of two electrons to the partially-filled landau level, just as for the case of weak back-scattering.

If the number of quasiholes is odd, there is no even-odd alternation in $E_N$. The period oscillations is then $\Delta A = 2\Phi_0/B$. Again, as in the weak back-scattering case, there is no oscillation with period $\Delta A = 4\Phi_0/B$.

Different behavior for even and odd quasihole number is a consequence of the particular non-abelian statistics of the Moore-Read state.
Weak back-scattering: $V_{12} \propto |t_1 + t_2 e^{2\pi i \Omega}|^2$, with $\delta \Omega = \delta A B/4\Phi_0$, if the qh number is even.
If central region contains an odd number of localized quasiparticles, this interference term is absent. Then leading interference term varies as

\[ \text{Re} \left[ t_1^* t_2 e^{2\pi i \Omega} \right]^2. \] (Period corresponds to an area containing two flux quanta, rather than four.)
Conclusion

Number parity is a subtle and fascinating issue in fractional quantized Hall states, as well as in superconductors.

There are still many aspects of this subject which are poorly understood.

I hope Vinay may find some time to think about these questions, now that he is “retired.”
Nobel Laureate Walter Kohn will present a new documentary on solar electricity, created in collaboration with Nobel Laureate Alan Heeger and host/narrator John Cleese.

The film is a scientific morality tale: how, starting from the most pure and basic science, through stages of brilliant applied science and engineering, there emerges a promising multi-billion dollar technology to help deal with one of the great challenges of our time: energy. That is, finding economically realistic, clean and safe energy sources to replace diminishing cheap fossil fuels at a time when energy demands of the developing world continue to grow rapidly.

Free and Open to the Public
Saturday, June 16, 2007
4:35pm
Schwartz Auditorium
Rockefeller Hall
Symposium remarks about P-G de Gennes

Thank you, distinguished guests, for voyaging to Ithaca, not the easiest odyssey, for this event! I consider it a great personal compliment. Thank you also to the local participants for sacrificing a typically beautiful Ithaca Saturday to attend.

My purpose now, however, is to remember someone who is not here. As most of you know, Pierre-Gilles de Gennes died on May 18. He had intended to be a speaker, and for most of the year was listed on the web site of this symposium, followed by the words (health permitting). Communications from him during the year had preprints on diverse topics attached to them. In mid-April he sent what has now to be viewed as a poignantly wishful acceptance, and the title: "High Tc superconductors: what makes copper special." On May 11, I received an email from him saying that he was too unwell to travel. A week later he was gone.

This is not the time or place for an extended eulogy. All of you knew him as an astonishingly versatile and inventive physicist; some of us as a colleague and friend. He had enough self-confidence to make lesser talents feel that they had something important to contribute to our shared enterprise. I, personally, am saddened and shaken by his all-too-early demise. Shall we stand in silence for a few moments to remember a great and creative person?

________________________________________________________

Thank you. [I will send a note to his widow informing her of this act.]

Vinay Ambegaokar
June 16, 2007
**Symposium Attendees**

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<thead>
<tr>
<th>Name</th>
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<td>Alexis Baratoff</td>
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<td>Chava Brender</td>
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<td>Aashish Clerk,*</td>
<td>McGill University</td>
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<td>Theja De Silva</td>
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<td>Ulrich Eckern</td>
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<td>Allan Griffin,*</td>
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<td>Bertrand Halperin</td>
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<td>Louis Hand</td>
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<td>Joseph Serene,*</td>
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<td>Eric Siggia</td>
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<td>Robert Silsbee</td>
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<td>Ravindra Sudan</td>
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<td>Stephen Teitel</td>
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<td>André-Marie Tremblay</td>
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<td>Xiaoliang Zhu</td>
<td>Wells College and Cornell University</td>
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*Organizing Committee
Attendees

50 Years of Condensed Matter Physics:
A Symposium on the Occasion of Vinay Ambegaokar’s Retirement

Saturday, June 16, 2007
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