Class question 1.3: two-variable Gaussian

Consider the probability density

\[ P(x, y) = \frac{1}{N} e^{-\frac{(x^2 - 2axy + y^2)}{2}} \]  \hspace{1cm} (1)

(a) Can you put (1) into the form

\[ P(x') P(y') \]  \hspace{1cm} (2)

where \( x' \) and \( y' \) are linear combinations of \( x \) and \( y \)?

Hint: the exponent in (1) is a linear combination of \((x + y)^2\) and \((x - y)^2\).

Don't worry, here, about the normalization factor \( N \).

(b) Using what you know for a single Gaussian, what is \( \langle x^2 \rangle \) for distribution (1)?

Hint: you'll need to write \( x \) in terms of \( x' \) and \( y' \).