Numerical Schwinger Boson Approach to the Bethe lattice antiferromagnet at percolation

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Variational mean field theory, SU(2) → bosons (Arovas & Auerbach PRB 1988)

# of boson flavors $N$ to be arbitrarily large → allows an expansion in $1/N$ to generate a mean field Hamiltonian

Resulting $H_{MF}$ is spin rotationally invariant

Typical applications: studying Spin Hamiltonians on frustrated lattices, but translationally invariant

Aim: Study Heis. NN AFM. $J \sum_{\langle ij \rangle} S_i \cdot S_j$ on non uniform systems
Mapping of SU(N) spins to Schwinger Bosons

**Physical Constraint:** Total Bosons/Site = NS
Large N Hamiltonian

- Variational Parameters
  → Bond variables $Q_{ij}$, 1 for every bond $\langle ij \rangle$
  → Lagrange multipliers $\lambda_i$
  → Condensate field $\beta_i$ to check for LRO

- The mean field Hamiltonian is given by

$$\mathcal{H}_{AFM} = \mathcal{H}_{exc.bosons}(Q_{ij}, \lambda_i) + E_{cond.} + \mathcal{H}_{constraint}$$

$$\mathcal{H}_{exc.bosons}(Q_{ij}, \lambda_i) = \sum_{\langle ij \rangle} \left( Q_{ij} a_i a_j + Q_{ij}^* a_i^\dagger a_j^\dagger \right) + \sum_{\langle ij \rangle} \frac{|Q_{ij}|^2}{j}$$

$$E_{cond.} = \sum_i \lambda_i |\beta_i|^2 + \sum_{\langle ij \rangle} \left( Q_{ij} \beta_i^* \beta_j^* + Q_{ij}^* \beta_i \beta_j \right)$$

$$\mathcal{H}_{constraint} = \sum_i \lambda_i \left( \sum_{flavors} \left( a_i^\dagger a_i \right) - NS \right)$$
Numerical Objective

Objective: Make a combined cost function to implement constraints

Fixed Boson Constraint

\[ \sum_{i=1}^{N_s} \lambda_i (n_b - N_S) \]

\[ \langle \text{......} \rangle = N_S \]
**Numerical Objective**

Objective: Make a combined cost function to implement constraints

- **Fixed Boson Constraint**
  \[
  \sum_{i=1}^{N_S} \lambda_i (n_b - NS)
  \]

- **Minimizing** $E_{MF}$ w.r.t

  Quantum Disordered Phase, all $\beta_i = 0$
  
  Long Range Order, non zero $\beta_i$
Numerical Objective

Objective: Make a combined cost function to implement constraints

Fixed Boson Constraint

\[ \sum_{i=1}^{N_S} \lambda_i (n_b - N_S) \]

Quantum Disordered Phase, all \( \beta_i = 0 \)

Long Range Order, non-zero \( \beta_i \)

Minimizing \( E_{MF} \) w.r.t. \( Q_{ij} \)

Self consistent condition \( \langle a_i a_j \rangle = Q_{ij} \)
Numerical Implementation

Optimization using Nelder Mead

Cost = \( w_{\lambda} \lambda_{\text{cost}} + w_{\beta} \beta_{\text{cost}} + w_{Q} Q_{ij} \) \( \text{cost} \)

Nelder Mead – non local algorithm
Does not use derivatives

Algo. scaling \( \sim \tau_{\text{steps}} N_s^3 \)

Difficulties!

# Parameters scales \( \sim \) system size \( N_s \)
Many local minima, similar in energy
Need to look at correlations / cost functions
Bond centered vs. Site centered Bethe lattice

<table>
<thead>
<tr>
<th>Bond centered pure Bethe lattice of coordination 3</th>
<th>Site centered pure Bethe lattice of coordination 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balanced $N_{\text{even}} = N_{\text{odd}}$</td>
<td>Unbalanced $N_{\text{even}} \neq N_{\text{odd}}$</td>
</tr>
</tbody>
</table>

Kumar et al. arXiv:1111.1442
Spin 1/2 Heisenberg model
Gapped spectrum, exp. decay corr.
Local Imbalance on bond centered Bethe lattice

Monomers due to local sublattice imbalance
Local Imbalance on bond centered Bethe lattice

Monomers due to local sublattice imbalance

Role of monomers also highlighted by Wang & Sandvik PRB 2006, 2010
Results: Effect of Local Imbalance on Pure Bethe lattice

\[ E = \frac{S(S+1)}{2N_S \chi_2} \]

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Branched Cluster DMRG calculation by HJC

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Numerical Schwinger Boson Approach to the Bethe lattice
Results: Effect of Local Imbalance on Pure Bethe lattice

$$E = \frac{S(S+1)}{2N_s\chi_2}$$

DMRG data

High energy rotor
Low energy rotor

$S^* = 10$

Monomers $\rightarrow$ Effective Quantum Rotor system of their own!

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Numerical Schwinger Boson Approach to the Bethe lattice
Long Range Order on the pure Bethe lattice at $T = 0$

Spin–spin correlations from DMRG and condensed SBMFT: $N_s = 126$
Long Range Order on the pure Bethe lattice at $T = 0$

- Lowest Spinon mode frequency $\omega_0 \sim 1/N_s \implies$ Gapless in thermodynamic limit (consistent with Correlations)
- Probe Role of monomers in enforcing Neel order
Relating Local condensate field $\beta_i$ to monomer locations

\[ \langle S_i \cdot S_j \rangle \text{ for } \langle ij \rangle \text{ from DMRG} \]

- $\langle S_i \cdot S_j \rangle$ pattern consistent with variation in local condensate field. Dangling spin density alternates across shells. Maximal at outermost.

Condensate Field on different sites for $N_s = 126$
Applying SBMFT to percolation clusters

Remove sites with probability $p_c = 1/2$
SBMFT predicted $\langle S_i \cdot S_j \rangle$ pattern is shown on the right. We can predict the locations of dangling spins!
- $\beta_i$ predicts higher amplitude on dangling sites!
- Condensate fraction highest on dangling sites! Hypothesis: Dangling spins enforce Neel order!
SBMFT correlations agree well with DMRG results, in addition it introduces the concept of a local condensate field $\beta_i$ which can highlight monomers

Hypothesis: Neel Order in the bond centered Bethe lattice is due to presence of such monomers

Need to study how Neel order is propagated in a dimer background for percolation clusters

Thank HJC for DMRG calculations, CLH and Michael Lawler for discussions and NSF and Cornell for support
Decouple the quartic Hamiltonian by introducing Hubbard Stratonovich Fields $Q_{ij}(\tau)$, one for every bond $\langle ij \rangle$.
Decoupling $\mathcal{H}_{AFM}$

Decouple the quartic Hamiltonian by introducing Hubbard Stratonovich Fields $Q_{ij}(\tau)$
Enforcing the $\lambda$ constraint

- Introduce a set of Lagrange multipliers $\{\lambda_i\}$
- Enforce constraint $\sum \langle a_i^m a_i^m \rangle = NS$
- Constraint is imposed only on average!
- Resulting $\Psi_{MF}$ is not a pure spin wave function. Contains fluctuations in boson density.
Tuning from Quantum disorder to LRO

SHORT RANGED CORRELATIONS
QUANTUM DISORDERED

S*
LONG RANGE ORDER
Mapping of SU(2) spins to Schwinger Bosons
Numerical Implementation
Results

Signature of Long Range Order

QUANTUM DISORDERED PHASE

ALL FLAVORS ARE DECOUPLED WITH DEGENERATE ENERGY LEVELS

SU(N) BROKEN SYMMETRY PHASE

FLAVOR ‘RED’ HAS A LOWERING OF ITS SINGLE SPINON
Pure Bethe pic
A. Author.

*Handbook of Everything.*

S. Someone.

On this and that.

*Journal on This and That.* 2(1):50–100, 2000.